

Crystal Growth: Physics, Technology and Modeling

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Lecture 6. Kinetic processes at surfaces

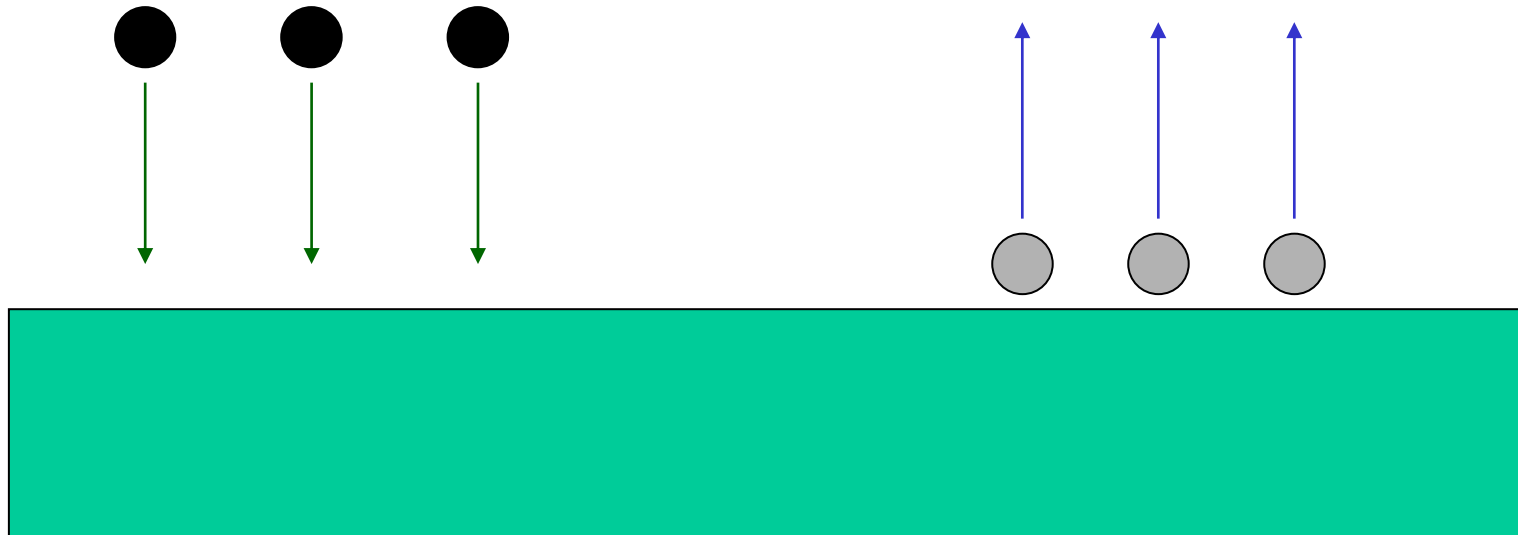
<http://www.unipress.waw.pl/~stach/cg-2024-25>

Synopsis

- **Adsorption and desorption**
- **Surface diffusion**
- **Step Properties**
- **Motion of a system of parallel steps**
- **2-d nucleation**
- **Screw dislocations role in surface dynamics**

Adsorption and desorption

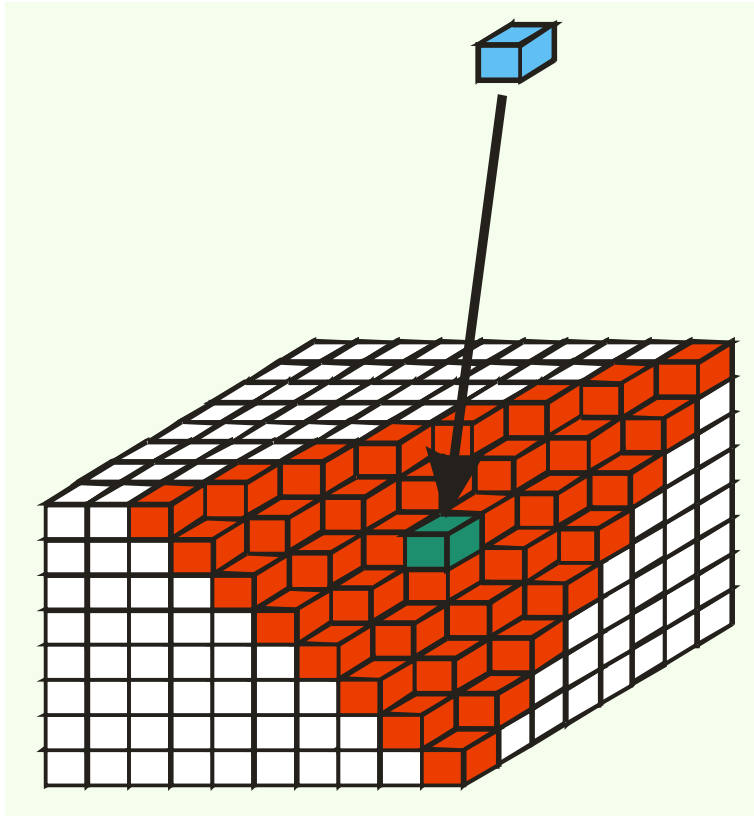
- **Adsorption** – attachment of species (molecules/atoms/radicals) at the solid/liquid surfaces
- **Desorption** – detachment of the species from solid/liquid surfaces



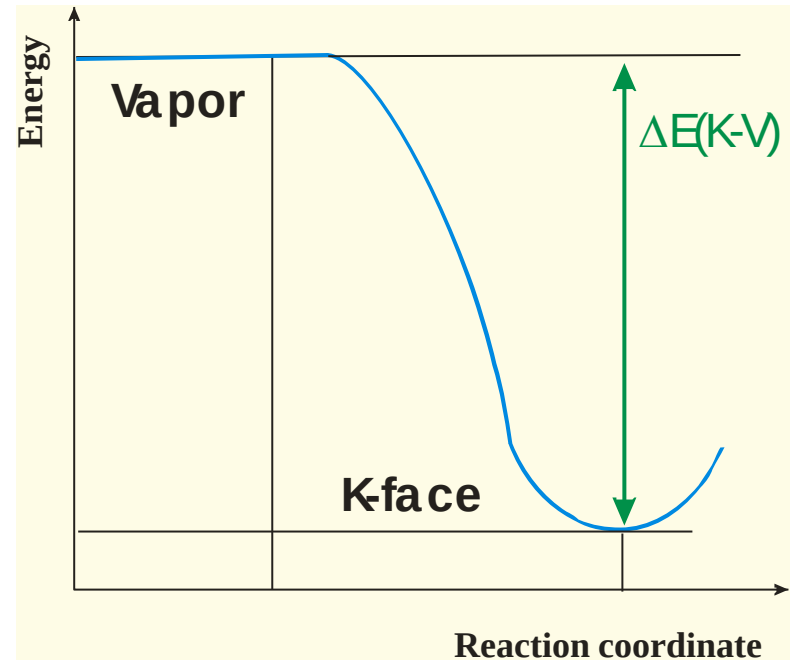
Desorption does not depend on the pressure in the vapor

Adsorption –Kossel model – K surface

- Atomic adsorption at K surface (atomically rough)



$$E_K = -3\phi$$



$$E_V = 0$$

K surfaces – fluxes and equilibrium

- Adsorption – site attachment rate R_{ads}

$$R_{ads} = \frac{p\zeta}{\sqrt{2\pi mkT}} = n\zeta \sqrt{\frac{kT}{2\pi m}}$$

- Desorption – detachment rate R_{des}

$$R_{des} = \nu \exp\left[-\frac{3\phi}{kT}\right]$$

- Equilibrium – fluxes are balanced R_{tot}

$$R_{tot} = R_{ads} - R_{des} = \frac{p_{eq}\zeta}{\sqrt{2\pi mkT}} - \nu \exp\left[-\frac{3\phi}{kT}\right] = 0$$

- Equilibrium – flux and pressure (will be used as reference value – i.e. zero supersaturation)

$$R_{eq} = \nu \exp\left[-\frac{3\phi}{kT}\right] \quad p_{eq} = \frac{\nu\sqrt{2\pi mkT}}{\zeta} \exp\left[-\frac{3\phi}{kT}\right]$$

K surface – growth rate

- Growth rate - v

$$v = I_{tot}d = [I_{ads}\zeta - R_{des}]d = \frac{p\zeta d}{\sqrt{2\pi mkT}} - vd \exp\left[-\frac{3\phi}{kT}\right]$$

d – atomic layer thickness (= a for Kossel crystal)

- Growth rate – in function of the pressure

$$v = \frac{[p - p_{eq}]\zeta d}{\sqrt{2\pi mkT}} = \frac{[p - p_{eq}]v_o}{\sqrt{2\pi mkT}}$$

$v_o = \zeta d$ (= a^3) – atomic volume

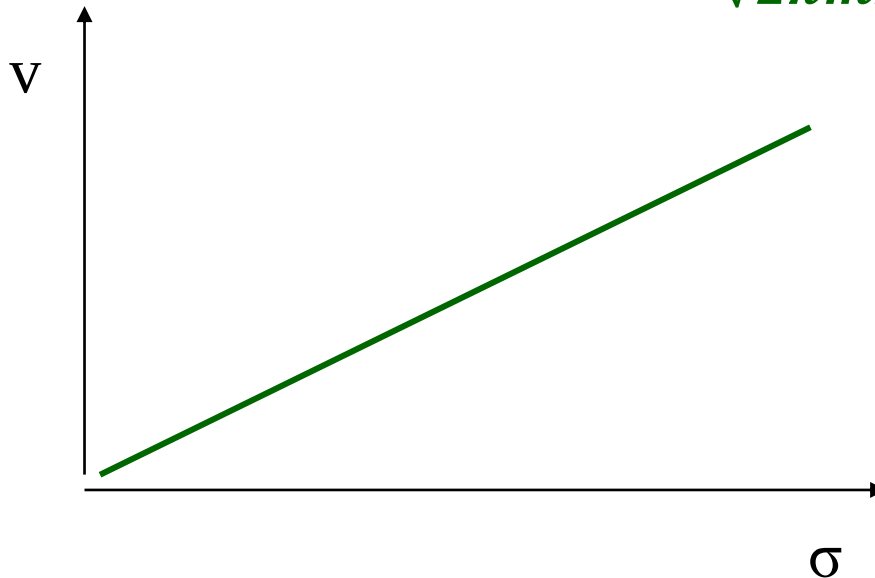
- Growth rate – in function of the supersaturation - Wilson- Frenkel law

$$v = \frac{p_{eq}v_o\sigma}{\sqrt{2\pi mkT}} = \sigma dv \exp\left[-\frac{3\phi}{kT}\right]$$

Wilson – Frenkel growth rate

- Growth rate v

$$v = \frac{p_{eq} v_o \sigma}{\sqrt{2\pi m k T}} = v_d \sigma \exp \left[-\frac{3\phi}{kT} \right]$$



$$v = 0.66 \text{ cm/s}$$

This is the upper bound for crystal growth velocity at:

i/ $\sigma = 0.1$

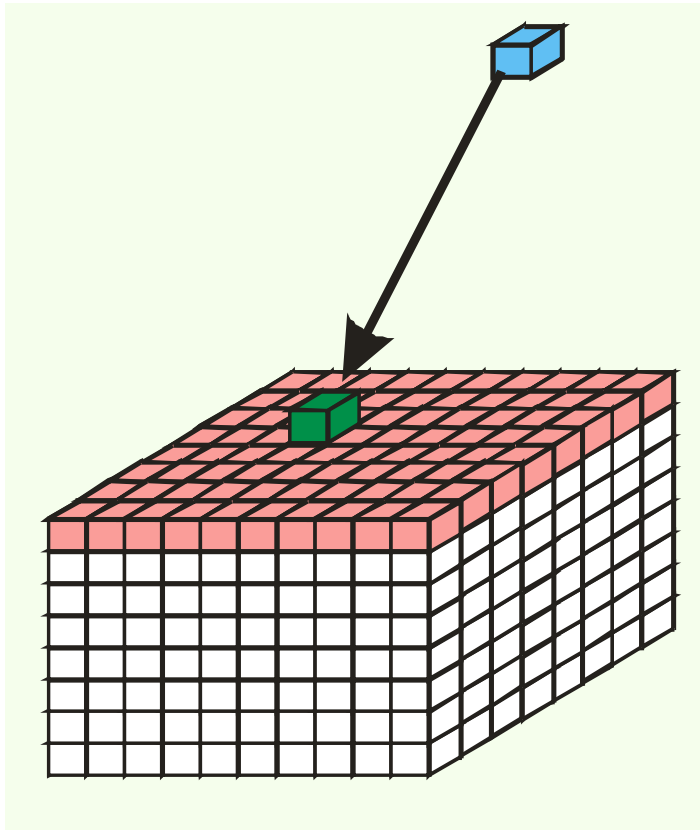
ii/ $p = 1 \text{ bar} = 10^5 \text{ N/m}^2$

iii/ $T = 298.15 \text{ K}$

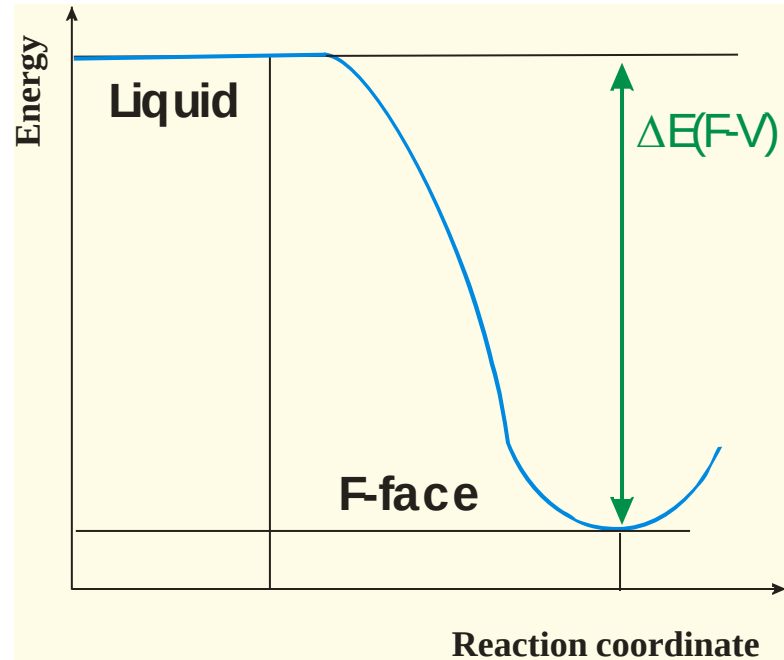
iv/ $m = m_{N_2} = 28 * 1.66 * 10^{-27} \text{ kg}$

Adsorption – F surface (Kossel model)

- Atomic adsorption at F surface (atomically flat)



$$E_F = -\phi$$



$$E_V = 0$$

Adsorption – at flat (F) surface

- Adsorption – site attachment rate R_{ads}

$$R_{ads} = I_{ads}\zeta = \frac{p\zeta}{\sqrt{2\pi mkT}} = n\zeta \sqrt{\frac{kT}{2\pi m}}$$

- Desorption – detachment rate R_{des}

$$R_{des} = \nu c_{sur}(p) \exp\left[-\frac{\phi}{kT}\right]$$

- F surface – vapor – crystal equilibrium (zero supersaturation) – fluxes are balanced I_{tot}

$$I_{tot} = \frac{p_{eq}}{\sqrt{2\pi mkT}} - \frac{\nu c_{sur-eq}}{\zeta} \exp\left[-\frac{\phi}{kT}\right] = 0$$

- Site occupation (coverage) c_{sur-eq}

$$c_{sur-eq} = \exp\left[-\frac{2\phi}{kT}\right]$$

Adsorption – at flat (F) surface- equilibrium with the crystal (K face)

- F surface – vapor in equilibrium with the crystal (K surface) – fluxes are balanced

$$I_{tot} = \frac{p_{eq}\zeta}{\sqrt{2\pi mkT}} - \nu c_{sur-eq} \exp\left[-\frac{\phi}{kT}\right] = 0$$

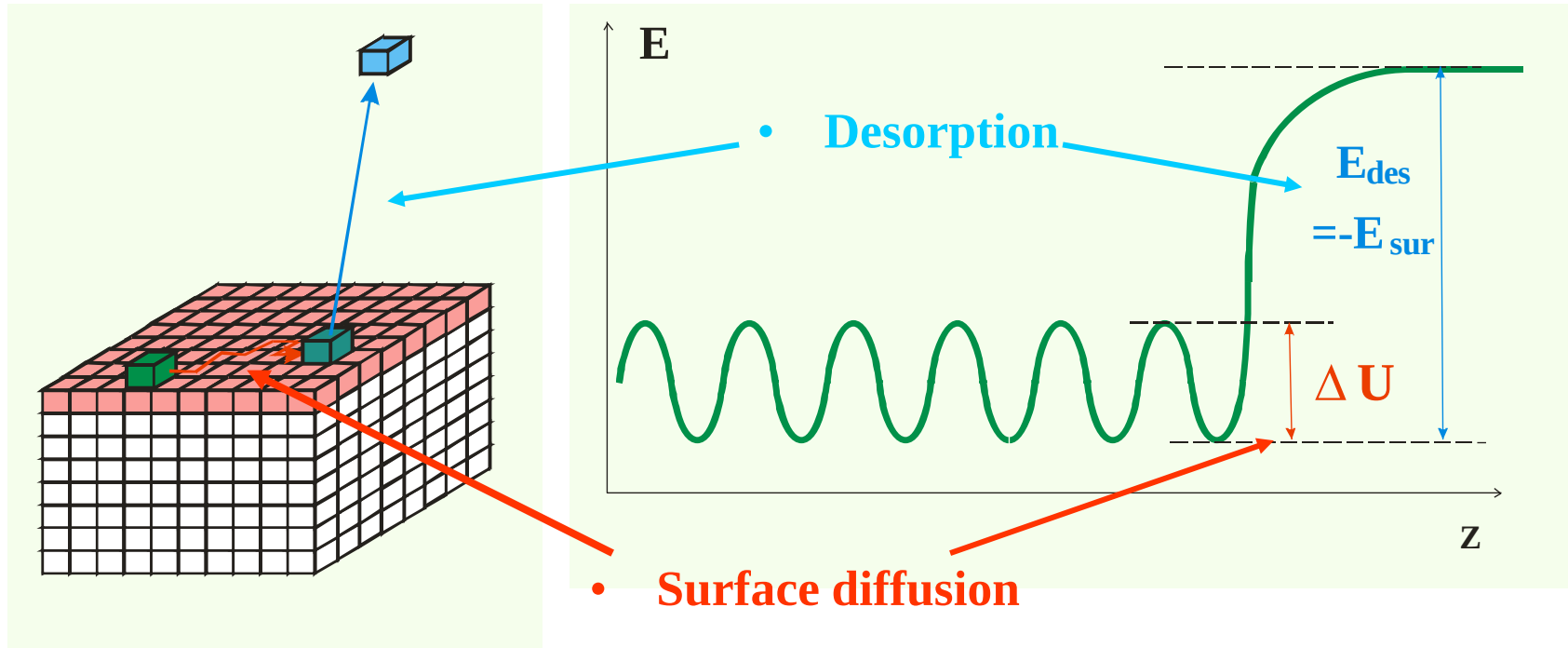
- Equilibrium with K face – flux and pressure

$$R_{eq} = \nu \exp\left[-\frac{3\phi}{kT}\right]$$

- Site occupation in equilibrium (zero supersaturation) with the crystal and with vapor:

$$c_{sur-eq} = \exp\left[-\frac{2\phi}{kT}\right]$$

Flat surface – surface diffusion and desorption



- Surface diffusion coefficient depends on the jump barrier U :

$$D = \frac{a^2 v}{4} \exp \left[-\frac{\Delta E_{diff}}{kT} \right] = \frac{a^2 v}{4} \exp \left[-\frac{U}{kT} \right]$$

Flat (F) surface - desorption

- Desorption – detachment rate R_{des}

$$R_{des} = \nu \exp \left[-\frac{E_{des}}{kT} \right] = \nu \exp \left[-\frac{\phi}{kT} \right]$$

- Surface residence time τ_{sur} – detachment rate

$$\tau_{sur} = \frac{1}{\nu} \exp \left[\frac{E_{des}}{kT} \right] = \tau_o \exp \left(\frac{\phi}{kT} \right)$$

The latter dependence is described for Kossel crystal F surface
i.e. for $E_{des} = \phi$.

F surface – surface diffusion length

- Surface diffusion length x_{sur} is determined using surface residence time τ_{sur}

$$\langle (\Delta r)^2 \rangle = 4D_{sur}t = 4D_{sur}\tau_{sur}$$

- Surface diffusion coefficient

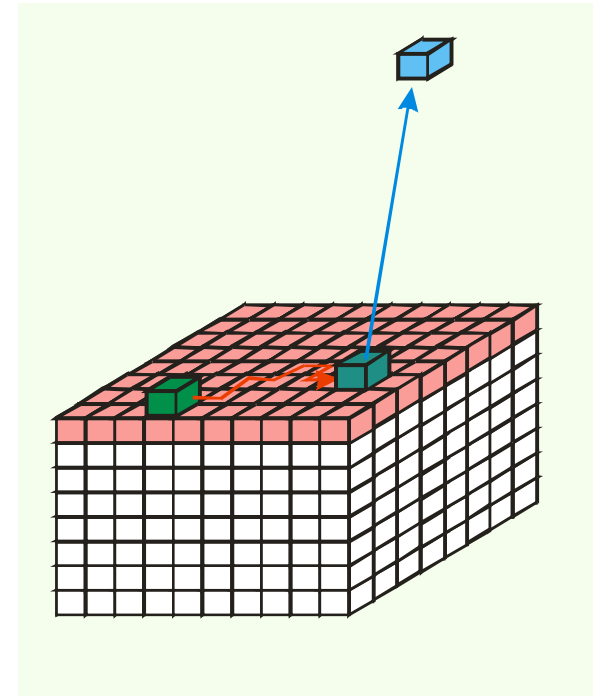
$$D_{sur} = \frac{a^2\nu}{4} \exp\left[-\frac{U}{kT}\right]$$

- Surface diffusion length x_{sur} is

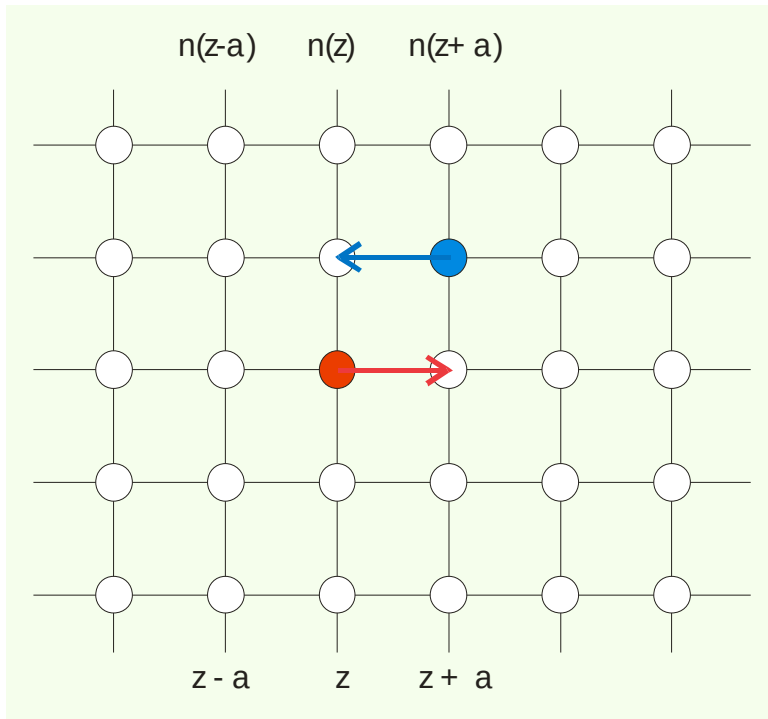
$$x_{sur} = \frac{a}{2} \exp\left(-\frac{E_{des} - \Delta E_{diff}}{2kT}\right)$$

- Kossel crystal

$$x_{sur} = \frac{a}{2} \exp\left(\frac{\phi - U}{2kT}\right)$$



Surface diffusion – chemical diffusion



- Average density of adsorbed species changes slowly in space:

$$n_{sur}(z+a) = n_{sur} + \delta n_{sur}$$

$$n_{sur}(z-a) = n_{sur} - \delta n_{sur}$$

i.e. over the distance a the change is small, and could be approximated by linear term.

- The velocity of the jump of the species to the **right** and **left** is

$$R(z \rightarrow z+a) = n_{sur}(z) v P(z \rightarrow z+a)$$

$$R(z+a \rightarrow z) = n_{sur}(z+a) v P(z+a \rightarrow z)$$

Surface diffusion (chemical) - Fick law

- Effective flux is the difference of these jumps

$$j_{sur} = R(z \rightarrow z + a) - R(z + a \rightarrow z)$$

- Linear expansion gives

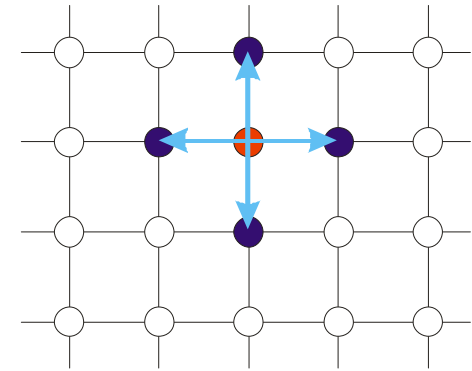
$$j_{sur} = - \left(\frac{dn_{sur}}{dz} \right) a v P(z \rightarrow z + a)$$

$$j_{sur} = - \frac{a^2 v}{4} \left(\frac{dn_{sur}}{dz} \right) = -D_{sur} \left(\frac{dn_{sur}}{dz} \right)$$

- General case – Fick law

$$\vec{j}_{sur} = -D_{sur} \nabla n_{sur} = -D_{sur} \eta \nabla c_{sur}$$

c_{sur} – coverage, i.e. fraction of sites that are occupied



$$P(z \rightarrow z + a) = \frac{1}{4}$$

Adsorbate dynamics

- Mass conservation law (next lecture – derivation)

$$\nabla \cdot \vec{j}_{sur} + I = 0$$

- Mass source for unit of surface area I – adsorption and desorption balance

$$I = I_{ads} - I_{des}$$

- Adsorbate dynamics equation

$$D_{sur} \Delta n_{sur} = I_{ads} - I_{des}$$

Sources – adsorption and desorption

- Vapor mass sources are calculated using vapor supersaturation σ_V :

$$\sigma_V = \frac{p - p_{eq}}{p_{eq}}$$

- Surface supersaturation, different from vapor supersaturation is;

$$\sigma = \frac{n_{sur} - n_{sur-eq}}{n_{sur-eq}} = \frac{c_{sur} - c_{sur-eq}}{c_{sur-eq}}$$

- Adsorbate density:

$$n_{sur} = \eta c_{sur} = \frac{c_{sur}}{\zeta}$$

- The effective source (i.e. from the vapor) is:

$$I = I_{ads} - I_{des} = \frac{p}{\sqrt{2\pi m k T}} - \frac{\nu c_{sur}}{\zeta} \exp\left[-\frac{\phi}{kT}\right]$$

Adsorbate dynamics - sources

- Laplace term is:

$$D_{sur}\Delta n_{sur} = D_{sur}\eta\Delta c_{sur} = \frac{D_{sur}\Delta(c_{sur} - c_{sur-eq})}{\zeta} = \frac{D_{sur}c_{sur-eq}\Delta\sigma}{\zeta}$$

- Vapor mass source is:

$$I_{ads} = \frac{p}{\sqrt{2\pi mkT}} = \frac{\nu\sigma_V c_{sur-eq}}{\zeta} \exp\left[-\frac{\phi}{kT}\right] = \frac{\sigma_V c_{sur-eq}}{\zeta\tau_{sur}}$$

- Desorption escape is:

$$I_{des} = \frac{\nu c_{sur}}{\zeta} \exp\left[-\frac{\phi}{kT}\right] = \frac{\nu\sigma c_{sur-eq}}{\zeta} \exp\left[-\frac{\phi}{kT}\right] = \frac{\sigma c_{sur-eq}}{\zeta\tau_{sur}}$$

Adsorbate dynamics - sources

- The basic equation is:

$$\frac{D_{sur}c_{sur-eq}\Delta\sigma}{\zeta} = \frac{\sigma c_{sur-eq}\tau_{sur}}{\zeta} - \frac{\sigma_V c_{sur-eq}\tau_{sur}}{\zeta}$$

- Transferred to:

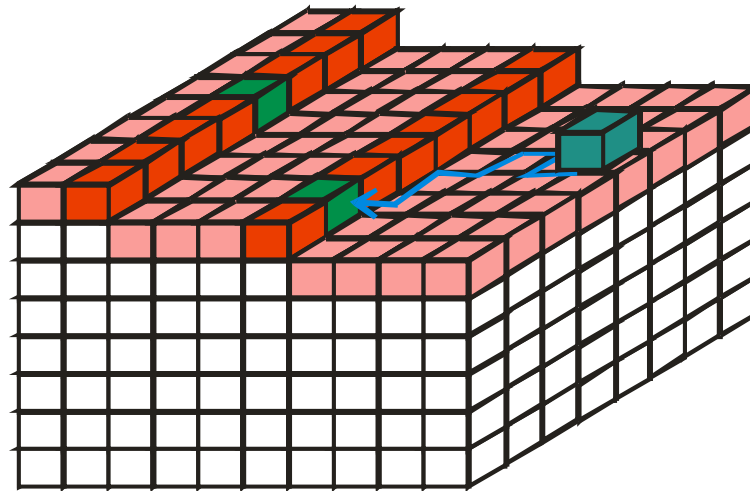
$$\frac{D_{sur}\Delta\sigma}{\tau_{sur}} = \sigma - \sigma_V$$

- The fundamental equation is:

$$x_{sur}^2\Delta\sigma = \sigma - \sigma_V$$

Vicinal (V) surfaces – steps and terraces

Surfaces consisting of steps and flat areas between (terraces) we call vicinal surfaces.

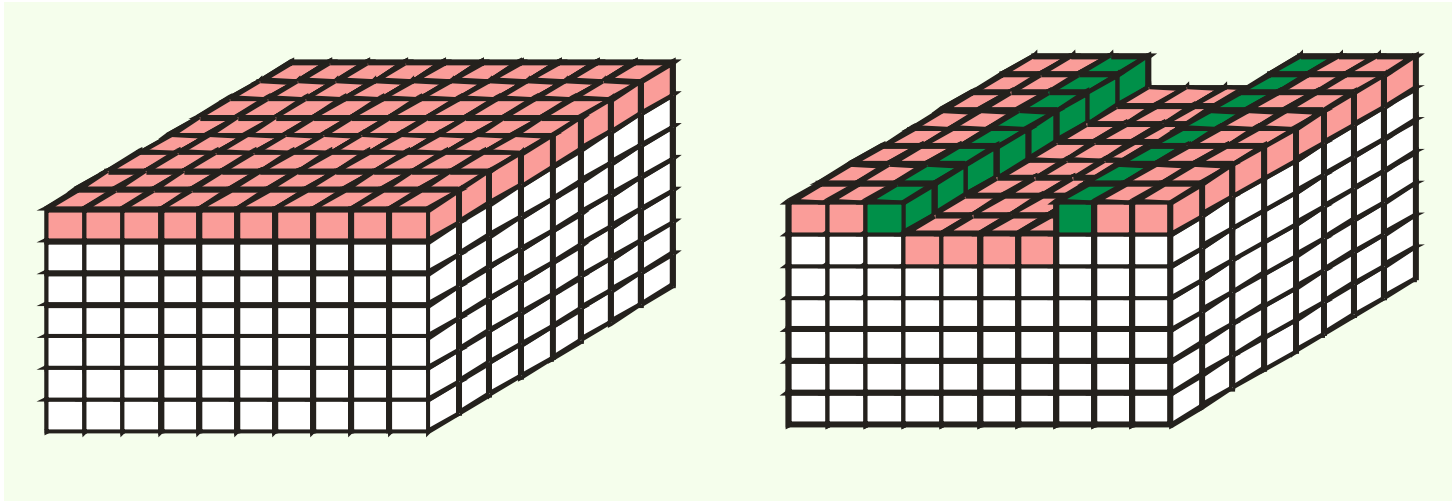


- The fundamental equation is governing adsorbate dynamics at terraces:

$$x_{sur}^2 \Delta \sigma = \sigma - \sigma_V$$

- Steps – boundary conditions?

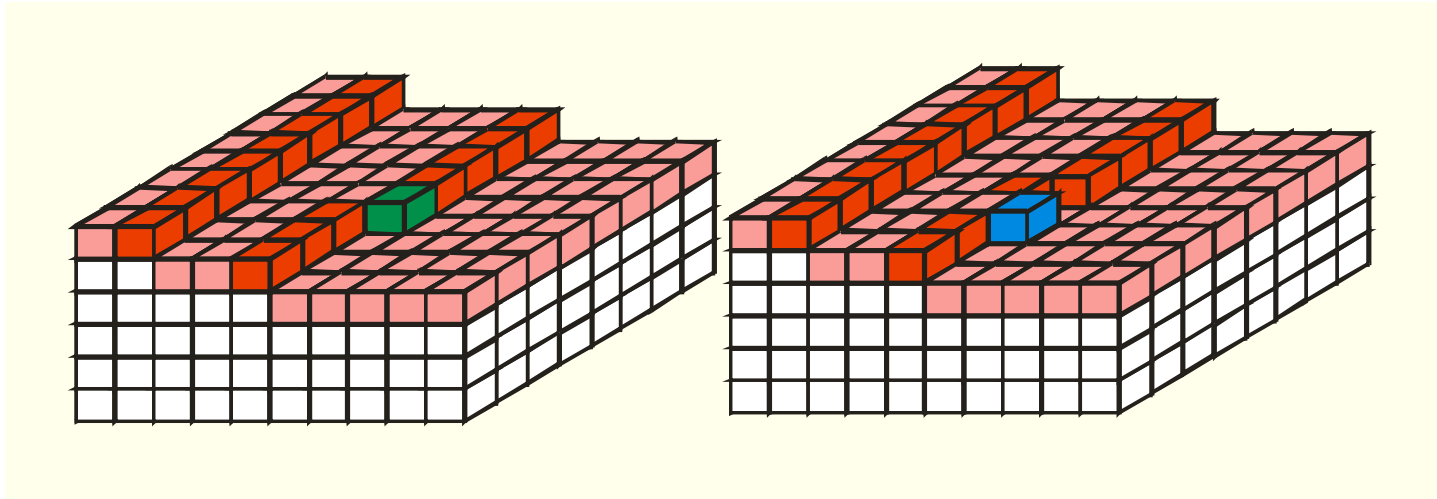
Energy of the steps



- **Atom row escape - creation of two steps**
- **Atom escape - two bonds are broken**
- **Step energy in a distance – half of the bond energy**

$$\gamma = \frac{\phi}{2}$$

Step- kink creation



- **Atom escape - creation of two kinks**
- **Atom escape - two bonds are broken**
- **Kink energy – half of the bond energy**

Microscopic structure of the steps

- **Atom escape - creation of two kinks**
- **Atom escape - two bonds are broken**
- **Kink concentration at the step:**

$$c_{kink} = P\left(\frac{E_{kink}}{2}\right) = \exp\left(-\frac{\phi}{2kT}\right)$$

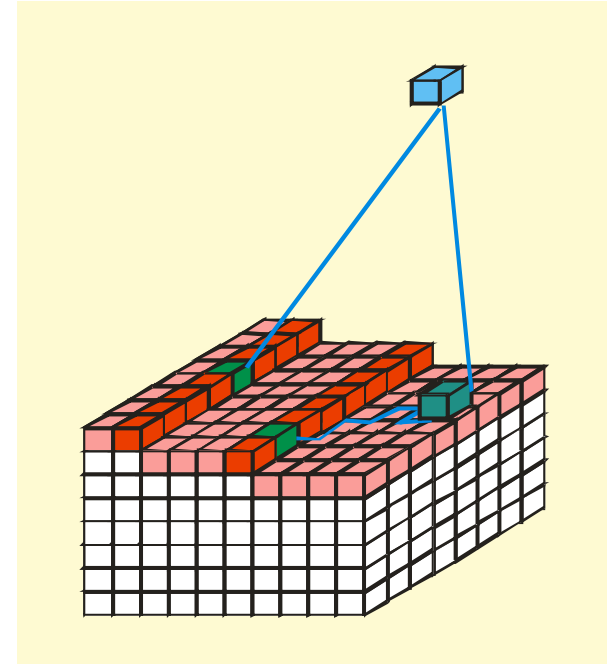
- **Kink concentration at the step is strongly temperature dependent. Nevertheless in typical growth experiment it is high. The distance between two kinks is**

$$l_{kink} = \frac{a}{2c_{kink}} = \frac{a}{2} \exp\left(\frac{\phi}{2kT}\right)$$

Steps – linear growth sources Burton, Cabrera Frank (BCF) model

Atoms/molecules are incorporated at the steps from:

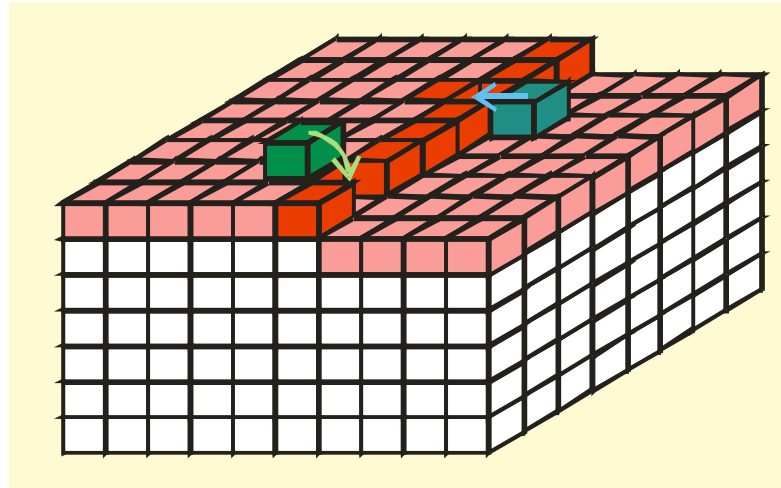
- Diffusion at terraces
- From the vapor directly



W.K. Burton, N. Cabrera & F.C. Frank, Philos. Trans. Roy. Soc. (London) A243 (1951) 299

Symmetry of the steps

- Symmetric steps - jump barrier from upper and lower are identical
- Asymmetric steps - jump barrier from upper and lower are different



- Step asymmetry – Schwoebel barrier

R.L. Schwoebel & E.J. Shipsey, J. Appl. Phys. 37 (1966) 3682

R.L. Schwoebel, J. Appl. Phys. 40 (1969) 614

Symmetric steps – simple model dynamics

- Jumps are equally probable from upper and lower terrace
- No additional energy barrier for terrace-step jump
- Simple model – supersaturation at the step vanishes
- Terrace dynamics

$$x_{sur}^2 \Delta \sigma = \sigma - \sigma_V \quad x_{sur}^2 \frac{d^2 \sigma}{dz^2} = \sigma - \sigma_V$$

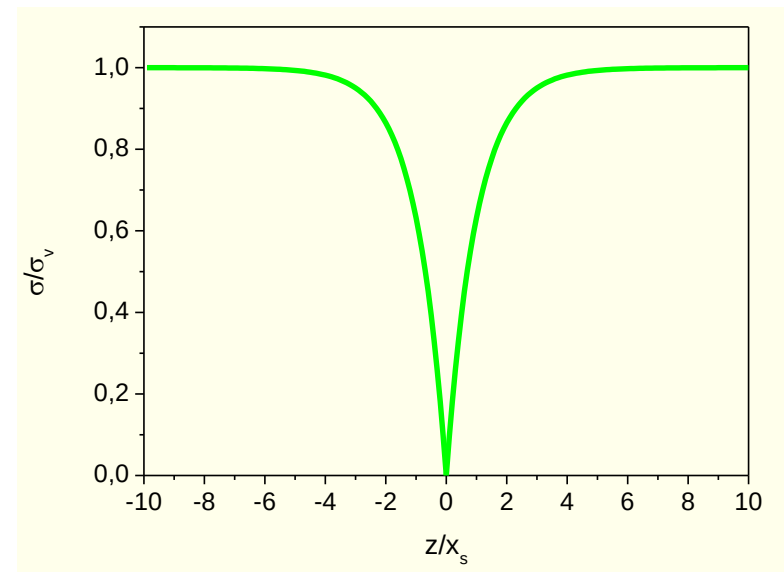
- Boundary condition at the step

$$\sigma = 0 \quad z = 0$$

- Solution

$$\sigma = \sigma_V [1 - \exp(-z/x_{sur})] \quad z > 0$$

$$\sigma = \sigma_V [1 - \exp(z/x_{sur})] \quad z < 0$$



Motion of single, symmetric step

- Adatom flux to the step I_{step}

$$I_{step} = 2j_{sur}(z = 0) = 2D_{sur}\eta c_{sur-eq}\nabla\sigma = \frac{2D_{sur}\eta c_{sur-eq}\sigma_V}{x_{sur}}$$

- Step site atom attachment frequency R_{step}

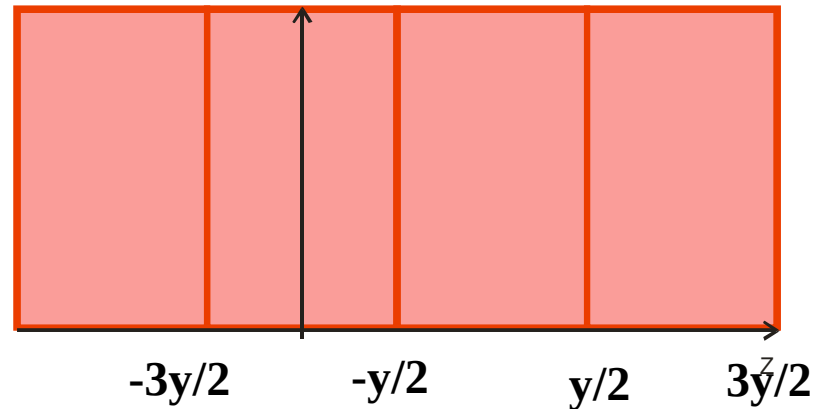
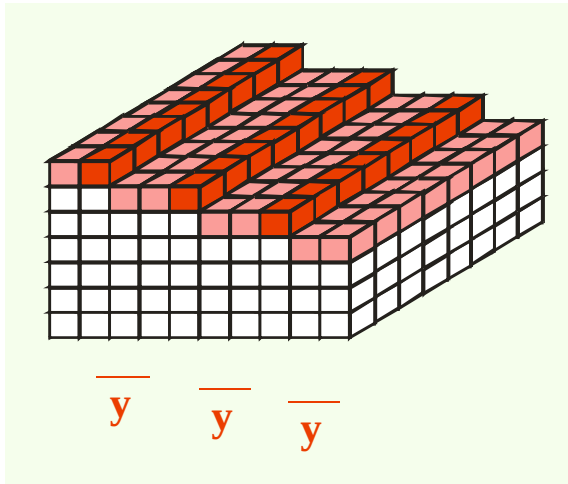
$$R_{step} = I_{step}a = \frac{2D_{sur}\eta c_{sur-eq}\sigma_V a}{x_{sur}}$$

- Step velocity v_{step}

$$v_{step} = R_{step}a = \frac{2D_{sur}\eta c_{sur-eq}\sigma_V a^2}{x_{sur}} = \frac{2x_{sur}c_{sur-eq}\sigma_V}{\tau_{sur}}$$

Parallel equidistant symmetric steps - dynamics

- Step distance y :



- Terrace dynamics

$$x_{sur}^2 \frac{d^2 \sigma}{dz^2} = \sigma - \sigma_V$$

- Boundary condition at the step

$$\sigma = 0 \quad z = \pm \frac{y}{2}, \pm \frac{3y}{2}, \pm \frac{5y}{2}, \dots,$$

- Solution (single terrace)

$$\sigma = \sigma_V \left[1 - \frac{\cosh(z/x_{sur})}{\cosh(y/2x_{sur})} \right]$$

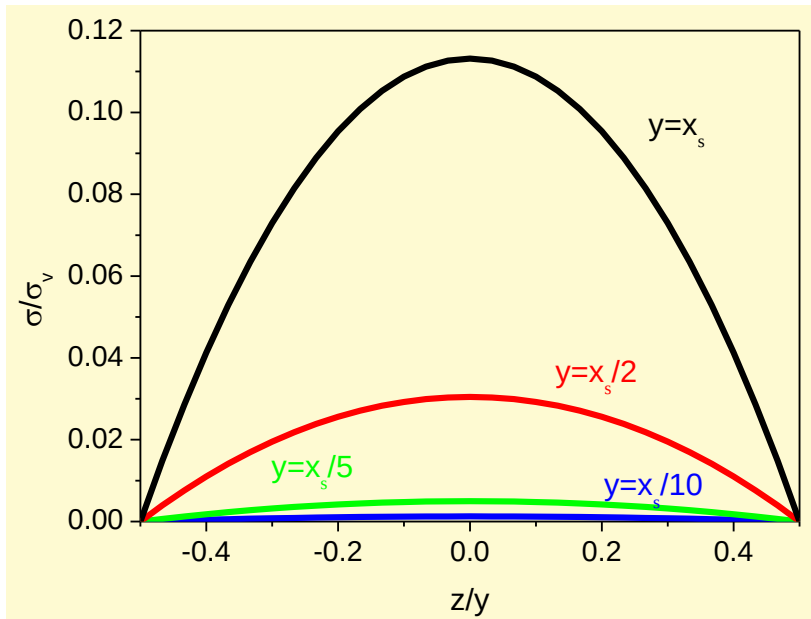
- Periodicity (multiple terraces)

$$\sigma = \sigma_V \left[1 - \frac{\cosh[(z - 2ny)/x_{sur}]}{\cosh(y/2x_{sur})} \right]$$

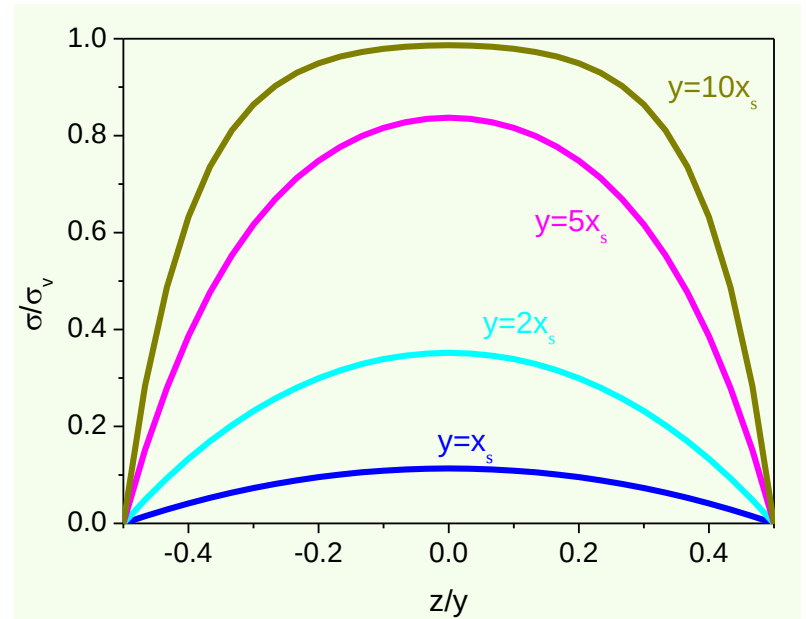
$$n = 0, \pm 1, \pm 2, \pm 3, \dots$$

Parallel equidistant symmetric steps - supersaturation

$$\sigma = \sigma_V \left[1 - \frac{csh(z/x_{sur})}{csh(y/2x_{sur})} \right] \quad -\frac{y}{2} \leq z \leq \frac{y}{2}$$



$$y \leq x_{sur}$$



$$y \geq x_{sur}$$

Parallel equidistant symmetric steps – step motion

- Adatom flux to the step I_{step}

$$I_{step} = 2D_{sur}\eta c_{sur-eq}\nabla\sigma = \frac{2D_{sur}\eta c_{sur-eq}\sigma_V}{x_{sur}} \tanh\left(\frac{y}{2x_{sur}}\right)$$

- Step site atom attachment frequency R_{step}

$$R_{step} = I_{step}a = \frac{2D_{sur}\eta c_{sur-eq}\sigma_V a}{x_{sur}} \tanh\left(\frac{y}{2x_{sur}}\right)$$

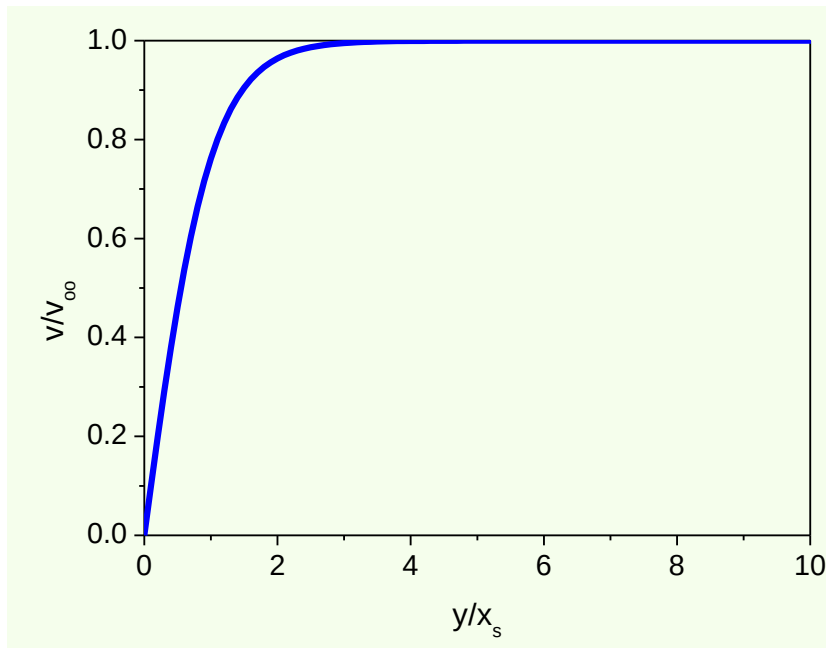
- Steps velocity $v_{step}(y)$

$$v_{step}(y) = R_{step}a = \frac{2x_{sur}c_{sur-eq}\sigma_V}{\tau_{sur}} \tanh\left(\frac{y}{2x_{sur}}\right)$$

Parallel symmetric steps – step motion – distance dependence

- Parallel steps velocity $v_{step}(y)$

$$v_{step}(y) = \frac{2x_{sur}c_{sur-eq}\sigma_V}{\tau_{sur}} \tanh\left(\frac{y}{2x_{sur}}\right)$$



- Single step velocity $v_{\infty} = v_{step}(\infty)$

$$v_{step}(\infty) = \frac{2x_{sur}c_{sur-eq}\sigma_V}{\tau_{sur}}$$

Small interstep distance – low velocity – diffusional repulsion of the steps

Asymmetric steps – simple model dynamics

- Jumps from upper and lower terrace have different barriers
- Simple model – supersaturation at the step vanishes
- Terrace dynamics

$$x_{sur}^2 \Delta \sigma = \sigma - \sigma_V \quad x_{sur}^2 \frac{d^2 \sigma}{dz^2} = \sigma - \sigma_V$$

- Boundary condition at the step – lower terrace

$$\sigma = 0 \quad z = 0_+$$

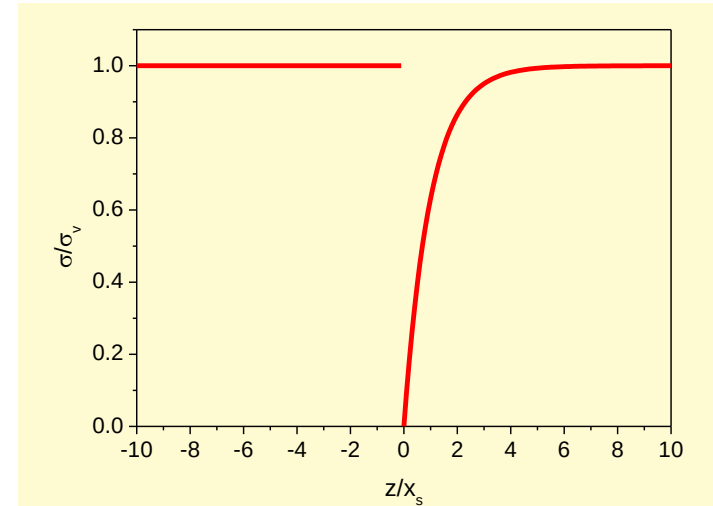
- Boundary condition at the step – upper terrace

$$j = 0 \quad z = 0_-$$

- Solution

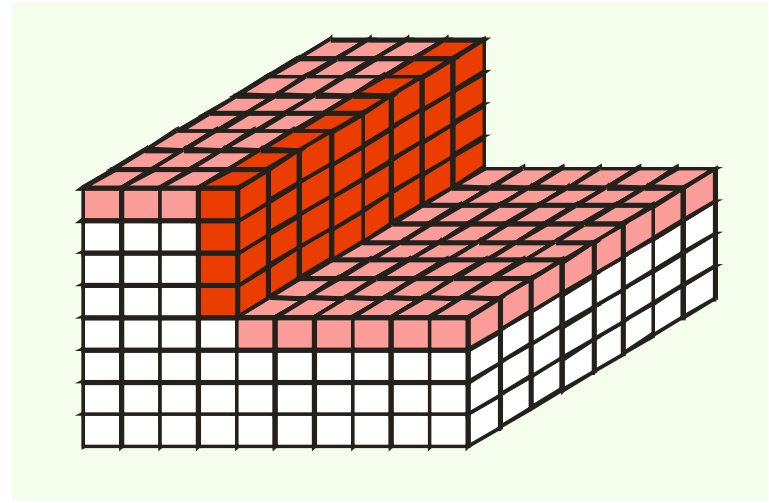
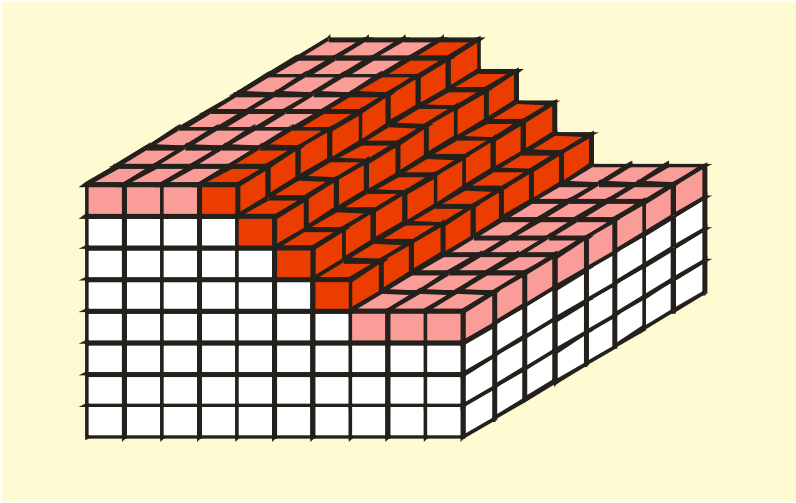
$$\sigma = \sigma_V [1 - \exp(-z/x_{sur})] \quad z > 0$$

$$\sigma = \sigma_V \quad z < 0$$



Parallel asymmetric steps - macrosteps

- **Macrostep** – stack of many atomic steps, at the same position



- **Macrosteps** – coalescence of a number of atomic steps
- Symmetric steps repel each other – no macrosteps creation tendency
- This conclusion is independent of the step orientation
- Additional factor can contribute to macrostep formation – impurities, kink poisoning, etc.

Steps – sources and geometry

Sources

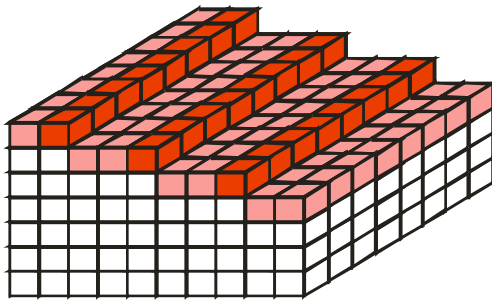
- **Surface preparation for epitaxy**
- **Two-dimensional nucleation**
- **Screw dislocation – continuous step source**
- **Other extended defects, such as stacking faults**

Geometry

- **Microstructure – equilibrium**
- **Macrostructure – step shape – purely nonequilibrium, history dependent**
- **Step height - purely nonequilibrium, history dependent**

Steps created by surface preparation

- **Surface misorientation** – small angle to low Miller indices surface e.g. 2° on GaN surface



$\overline{y}$ $\overline{y}$ $\overline{y}$

Sample size $l = 5 \text{ mm}$

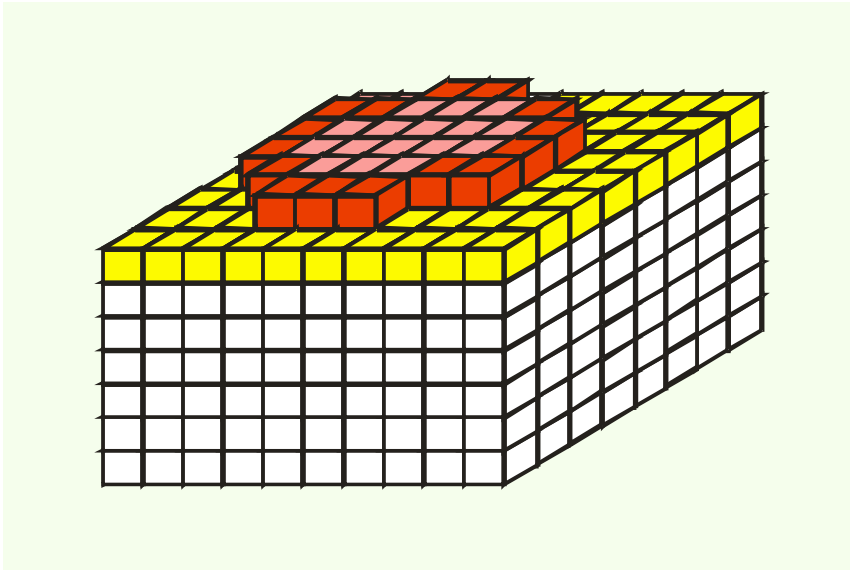
Layer thickness – d

$$d = l \operatorname{tg}(\alpha) = 172 \mu\text{m}$$

This is more than sufficient for typical optoelectronic device where the structure is 1 micron thick

Step source – 2-d nucleation

- **Fluctuation – emergence of finite island of a new atomic layer:**



- Edge energy of the length $a - \gamma$
- Single site area $s_0 = a^2$

- **Supersaturation**

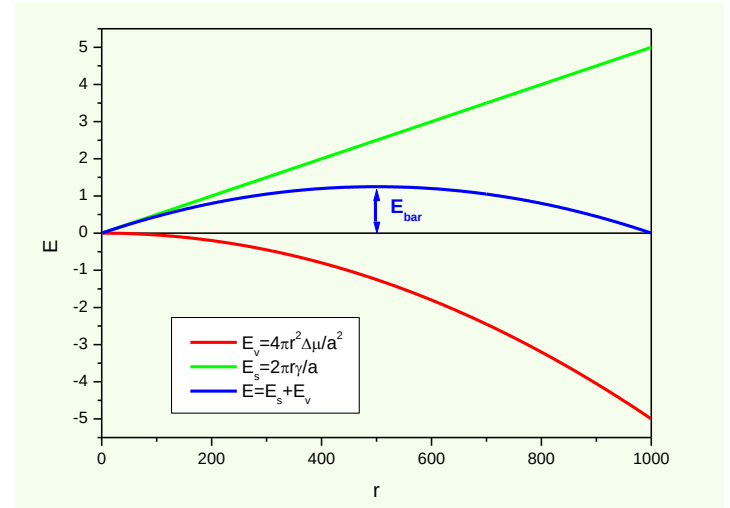
$$\sigma = \frac{\Delta\mu}{kT}$$

Nucleation energy barrier

- Total energy change in island creation:

$$\Delta E = \frac{\pi r^2 \Delta\mu}{a^2} + \frac{2\pi r \gamma}{a}$$

- Positive term is due to edge energy, $\gamma > 0$
- Negative term is due to bulk energy, $\Delta\mu < 0$



- Positive term dominates at small r , negative for large r , the maximum energy, at critical radius is called nucleation barrier

$$r_{crit} = -\frac{\gamma a}{\Delta\mu} = \frac{\gamma a}{|\Delta\mu|} = \frac{\gamma a}{kT\sigma} = \frac{\phi a}{2|\Delta\mu|} \quad \gamma = \frac{\phi}{2}$$

- Nucleation barrier

$$\Delta E_{bar} = \frac{\pi \gamma^2}{|\Delta\mu|} = \frac{\pi \phi^2}{4|\Delta\mu|}$$

2-d nucleation rate (Zeldowich)

- Nucleation rate – attempt frequency ν , number of sites at circumference N_s , and transition probability

$$R_{2d-nucl} = \nu n_c P = \nu \frac{2\pi r_{crit}}{a} \exp\left(-\frac{\Delta E_{bar}}{kT}\right)$$

- Nucleation rate – for single nucleus

$$R_{2d-nucl} = \frac{2\pi\nu\gamma}{|\Delta\mu|} \exp\left(-\frac{\pi\gamma^2}{kT|\Delta\mu|}\right) = \frac{\pi\nu\phi}{|\Delta\mu|} \exp\left(-\frac{\pi\phi^2}{4kT|\Delta\mu|}\right)$$

- Nucleation rate – for crystal surface A

$$\gamma = \frac{\phi}{2}$$

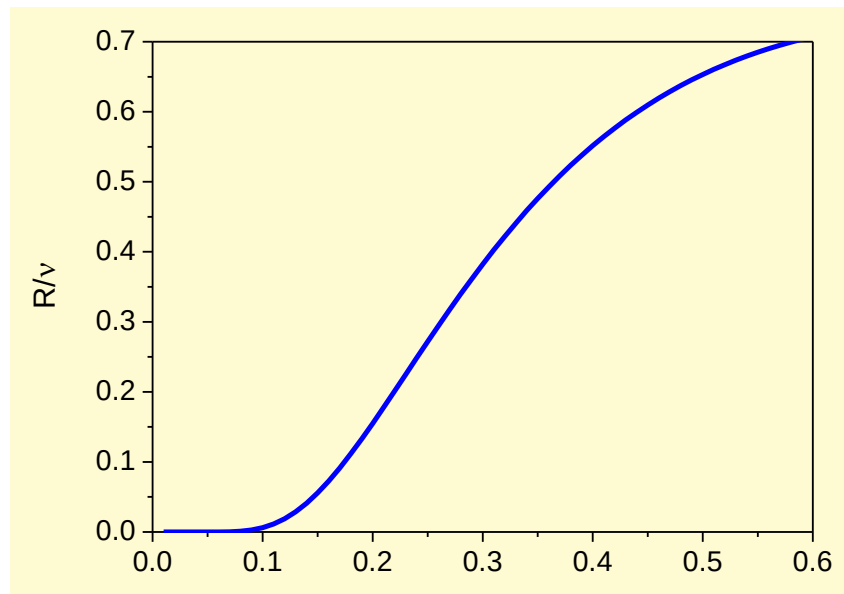
$$R_{2d-nucl}(A) = R_{2d-nucl} \frac{A}{\pi r_{crit}^2} = \frac{2\nu A |\Delta\mu|}{\gamma a^2} \exp\left(-\frac{\pi\gamma^2}{kT|\Delta\mu|}\right)$$

$$R_{2d-nucl}(A) = \frac{\nu A |\Delta\mu|}{\phi a^2} \exp\left(-\frac{\pi\phi^2}{4kT|\Delta\mu|}\right)$$

2-d nucleation rate – supersaturation dependence

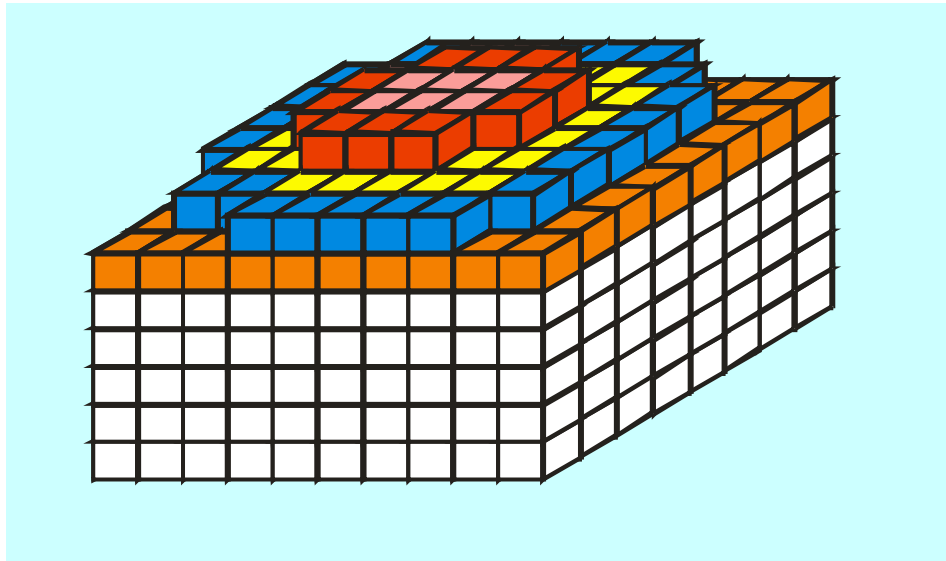
- Nucleation rate at the surface area A:

$$R_{2d-nucl}(A) = \frac{\nu A |\Delta\mu|}{\phi a^2} \exp\left(-\frac{\pi\phi^2}{4kT|\Delta\mu|}\right)$$



2-d nucleation – macrostep generation

- 2-d nucleation could be fast as compared to layer completion, - creation of macrosteps is possible



Growth – size increase of concentric circles

Growth of concentric circles

- Supersaturation at curved step is reduced by Gibbs-Thomson effect:

$$\sigma(r) = \sigma \left(1 - \frac{r_{crit}}{r} \right)$$

r_{crit} – critical nucleation radius (2d)

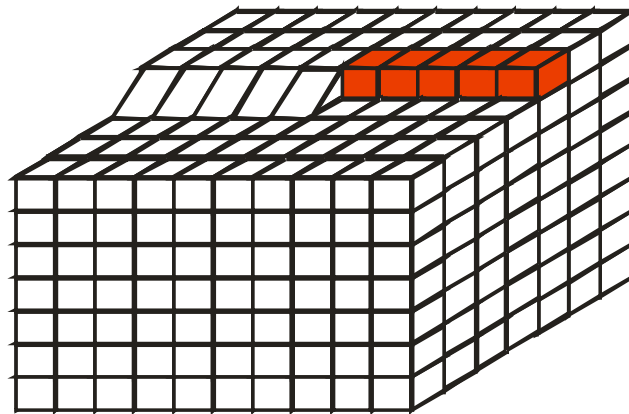
$$r_{crit} = -\frac{\gamma a}{\Delta\mu} = \frac{\gamma a}{|\Delta\mu|} = \frac{\gamma a}{kT\sigma} = \frac{\phi a}{2|\Delta\mu|}$$

- Velocity $v_{step}(y)$ of the curved steps, separated by the distance y :

$$v_{step}(y) = \frac{2D_{sur}\eta c_{sur-eq}\sigma_V a^2}{x_{sur}} \tanh\left(\frac{y}{2x_{sur}}\right) \left(1 - \frac{r_{crit}}{r}\right)$$

Step sources – screw dislocations

- **Dislocation atomic structure**

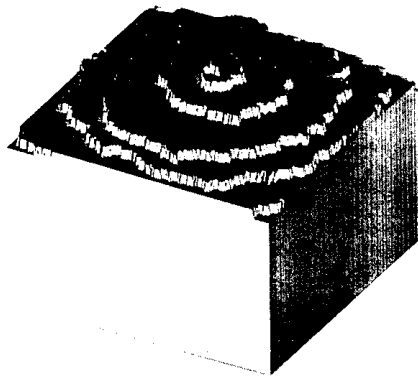
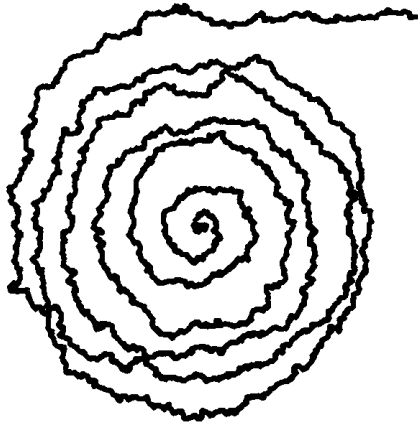


- **In equilibrium step is straight line.**

Supercooling – step is curved

- High temperature – Archimedean spiral

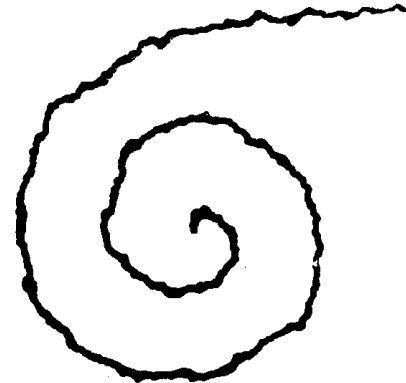
High supersaturation



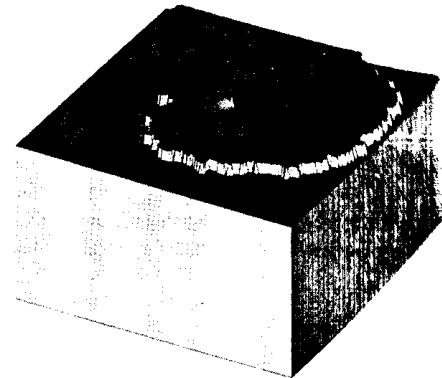
$$\sigma = \frac{\Delta\mu}{kT} = 0.4$$



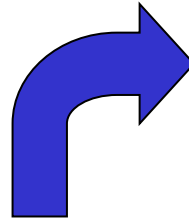
Low supersaturation



(a)



$$\sigma = \frac{\Delta\mu}{kT} = 0.1$$



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Step motion – spiral rotation – Archimedean spiral

- Step velocity depends on the curvature:

$$v_{step}(r) = v_{step} \left(1 - \frac{r_{crit}}{r} \right)$$

- Approximate solution- Archimedean spiral:

$$r(\theta) = 2r_{crit}\theta$$

- Angular velocity is:

$$\omega = \frac{v_{step}}{2r_{crit}}$$

- Distance between spiral arms is:

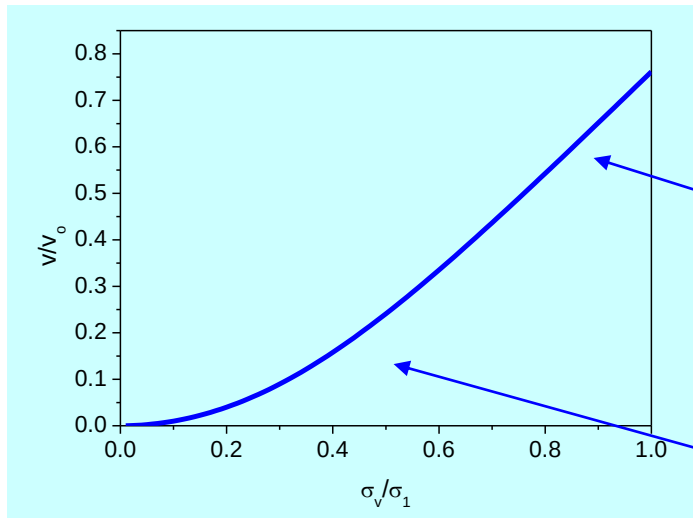
$$y_o = 2\pi r_{crit}$$

- Spiral shape is function of the supersaturation
- Spiral velocity depends on the surface diffusion rate

Spiral dislocation controlled growth rate

- Growth rate is equal to the angular velocity ω multiplied by layer thickness a :

$$v = \frac{\omega a}{2\pi} = \frac{v_{step} a}{2r_{crit}} = \frac{a\sigma_V^2}{\sigma_1 \tau_{sur}} \tanh\left(\frac{\sigma_1}{\sigma_V}\right) \cong \begin{cases} \frac{a\sigma_V^2}{\sigma_1 \tau_{sur}} & \sigma_V \ll \sigma_1 \\ \frac{a\sigma_V}{\tau_{sur}} & \sigma_V \gg \sigma_1 \end{cases}$$



$$\sigma_1 = \frac{\pi\gamma a}{2kTx_{sur}} = \left(\frac{a}{x_{sur}}\right) \left(\frac{\pi\phi}{4kT}\right)$$

linear

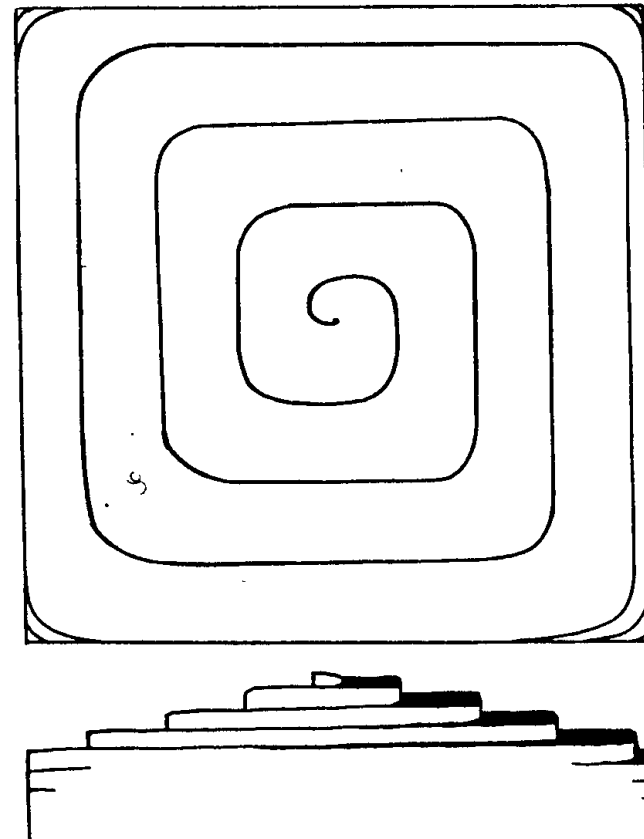
parabolic

Screw dislocation – growth pyramid.

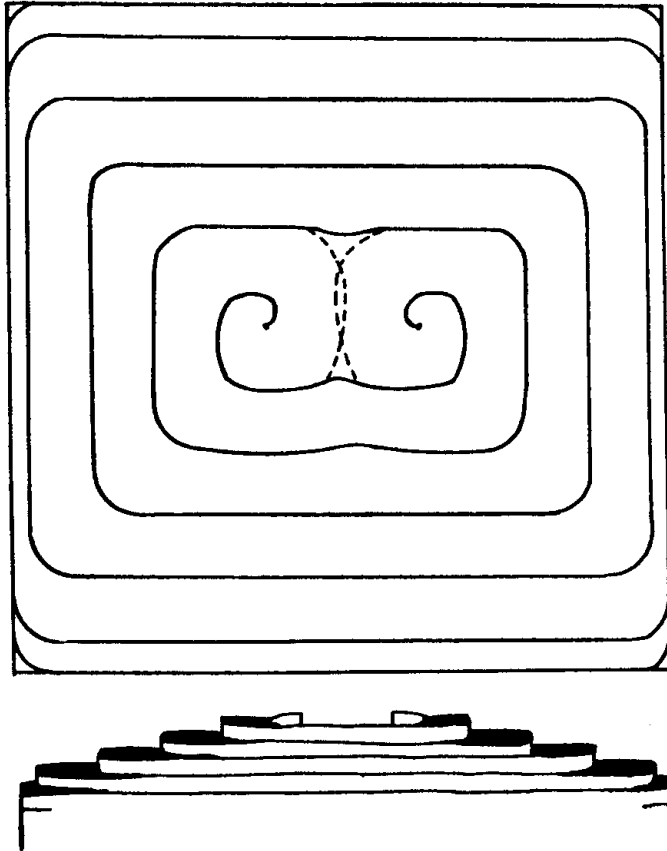
- Distance between spiral line is:

$$y_o = 4\pi r_{\text{crit}} =$$
$$\frac{\pi \gamma a}{k_B T \sigma_v} = \frac{\pi \varphi a}{k_B T \sigma_v}$$

$$\text{tg} \alpha = \frac{a}{y_o} = \frac{4k_B T \sigma_v}{\varphi}$$



Two dislocations of the opposite sign



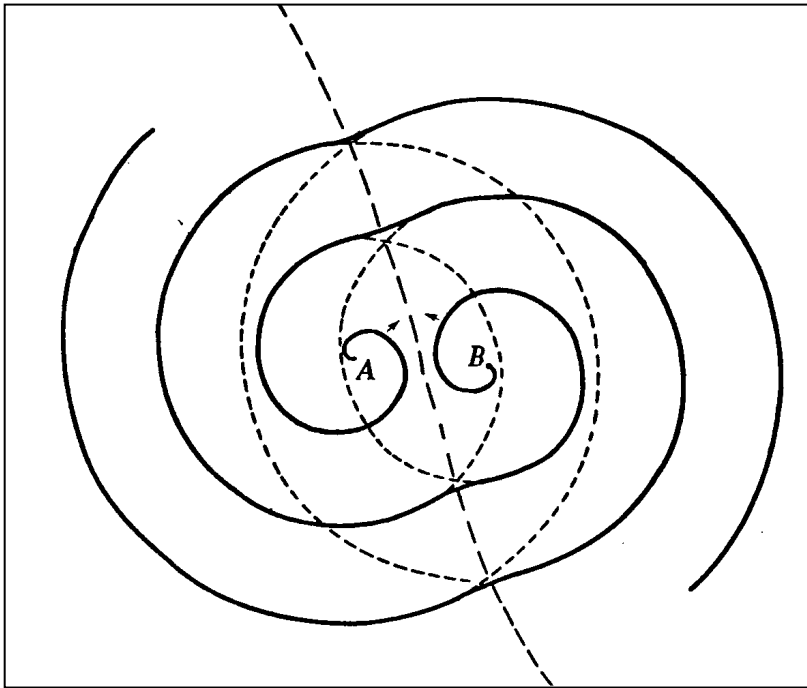
$L > r_{\text{crit}}$

L – distance between dislocations

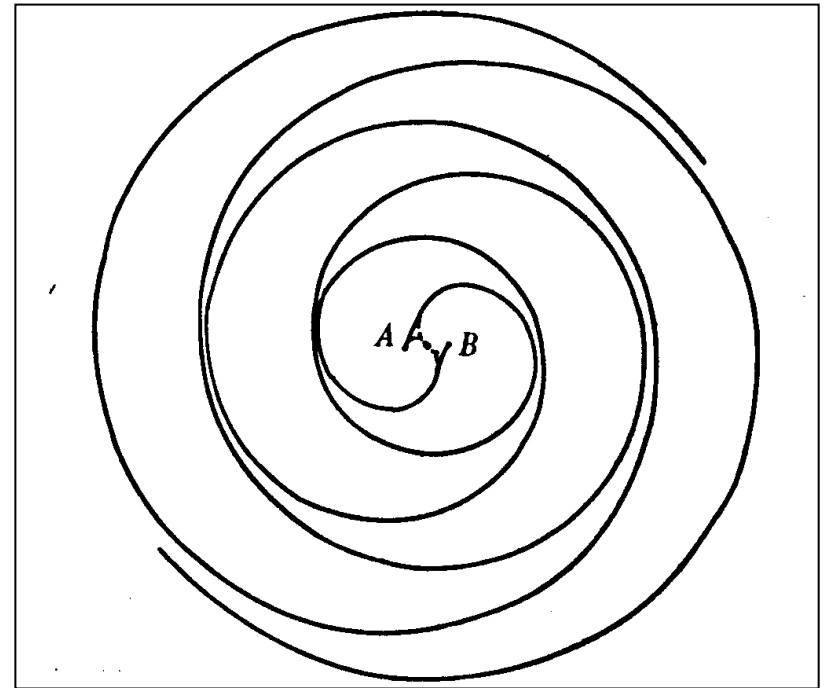


$L < r_{\text{crit}}$

Two dislocation of the same sign



$$L > r_{crit}$$



$$L < r_{crit}$$

Multiple dislocations of the same sign

