Crystal Growth: Physics, Technology and Modeling

Stanisław Krukowski & Michał Leszczyński
Institute of High Pressure Physics PAS
01-142 Warsaw, Sokołowska 29/37
e-mail: stach@unipress.waw.pl, mike@unipress.waw.pl

Zbigniew Żytkiewicz
Institute of Physics PAS
02-668 Warsaw, Al. Lotników 32/46
E-mail: zytkie@ifpan.edu.pl

Lecture 6. Kinetic processes at surfaces

http://www.unipress.waw.pl/~stach/cg-2021-22

Synopsis

- **Adsorption and desorption**
- **Surface diffusion**
- **Step Properties**
- **Motion of a system of parallel steps**
- **2-d nucleation**
- **Screw dislocations role in surface dynamics**

Adsorption and desorption

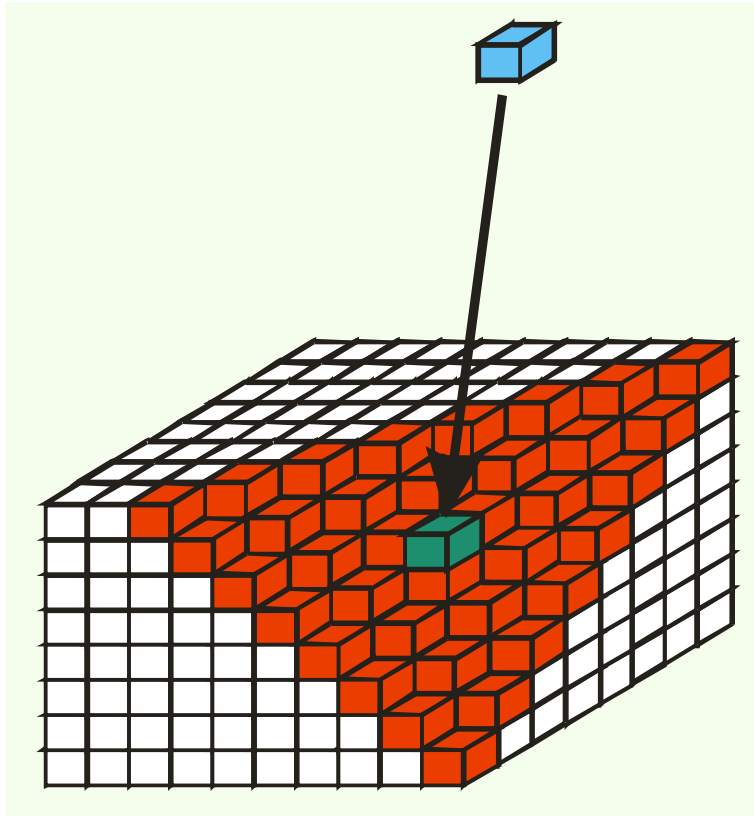- **Adsorption – attachment of species (molecules/atoms/radicals) at the solid/liquid surfaces**

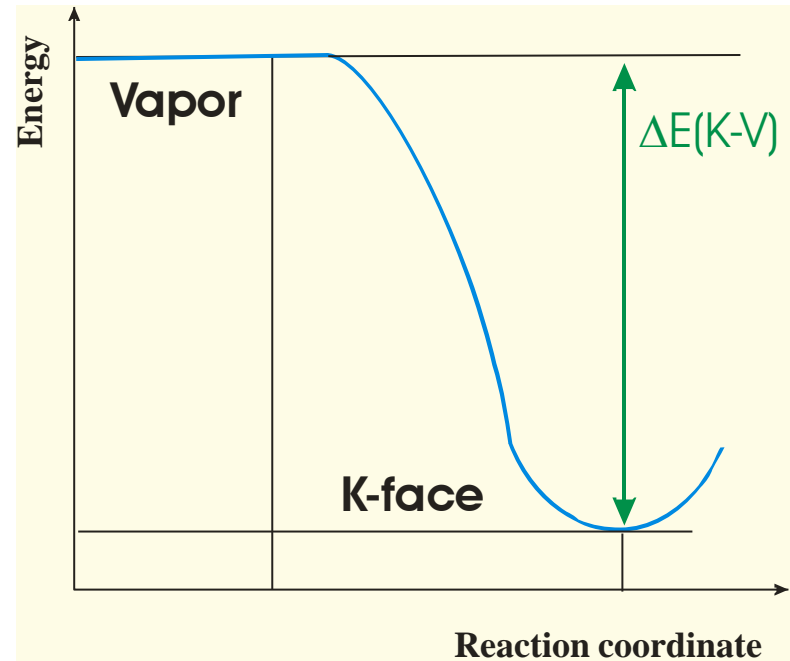- **Desorption – detachment of the species from solid/liquid surfaces**
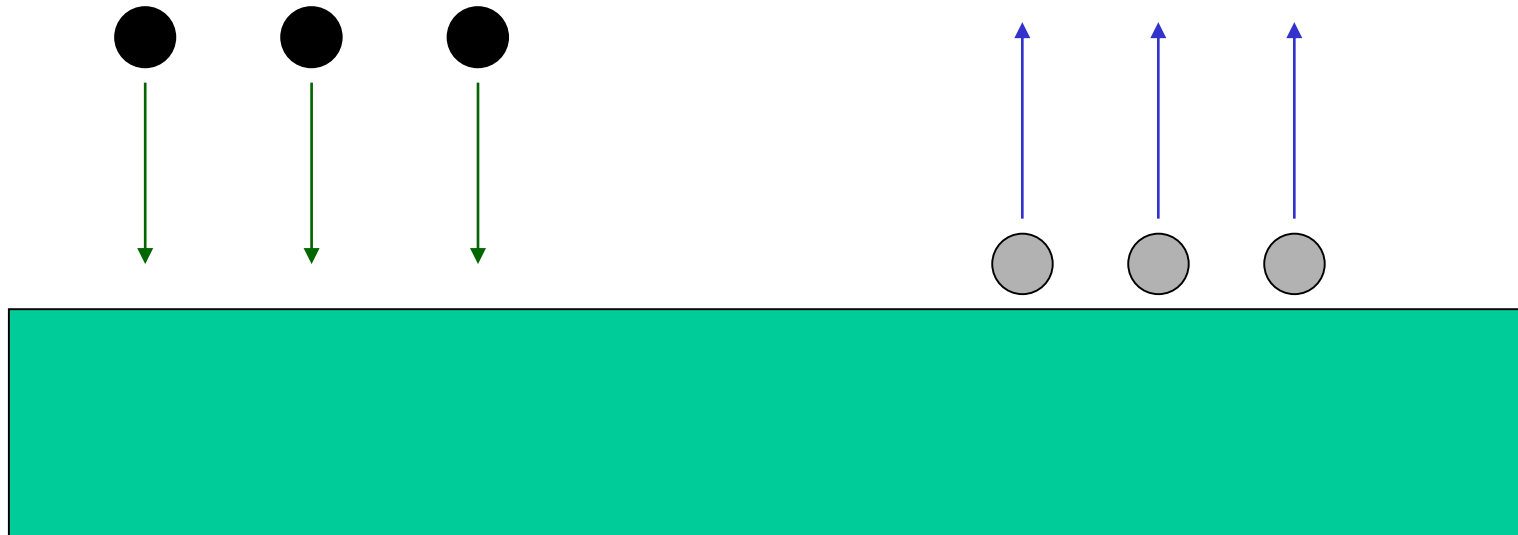
Desorption does not depend on the pressure in the vapor

Adsoprtion –Kossel model – K surface

- **Atomic adsoprtion at K surface (atomically rough)**

$$E_K = -3\phi$$

$$E_V = 0$$

K surfaces – fluxes and equilibrium

- **Adsorption – site attachment rate R_{ads}**

$$R_{ads} = \frac{p\varsigma}{\sqrt{2\pi mkT}} = n\varsigma\sqrt{\frac{kT}{2\pi m}}$$

- **Desorption – detachment rate R_{des}**

$$R_{des} = \nu\, exp\left[-\frac{3\phi}{kT}\right]$$

- **Equilibrium – fluxes are balanced R_{tot}**

$$R_{tot} = R_{ads} - R_{des} = \frac{p_{eq}\varsigma}{\sqrt{2\pi mkT}} - \nu\, exp\left[-\frac{3\phi}{kT}\right] = 0$$

- **Equilibrium – flux and pressure (will be used as reference value – i.e. zero supersaturation)**

$$R_{eq} = \nu\, exp\left[-\frac{3\phi}{kT}\right] \qquad p_{eq} = \frac{\nu\sqrt{2\pi mkT}}{\varsigma}\, exp\left[-\frac{3\phi}{kT}\right]$$

K surface – growth rate

- **Growth rate - v**

$$v = I_{tot}d = [I_{ads}\varsigma - R_{des}]d = \frac{p\varsigma d}{\sqrt{2\pi mkT}} - \nu d\ exp\left[-\frac{3\phi}{kT}\right]$$

d – atomic layer thickness (= a for Kossel crystal)

- **Growth rate – in function of the pressure**

$$v = \frac{[p - p_{eq}]\varsigma d}{\sqrt{2\pi mkT}} = \frac{[p - p_{eq}]v_o}{\sqrt{2\pi mkT}}$$

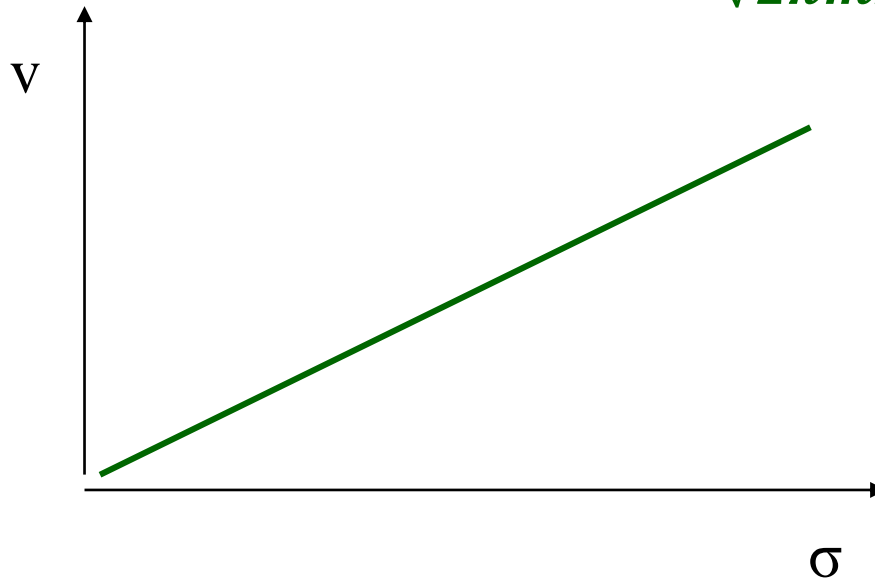$\boldsymbol{v_o} = \boldsymbol{\varsigma d}$ **(= a³) – atomic volume**

- **Growth rate – in function of the supersaturation - Wilson- Frenkel law**

$$v = \frac{p_{eq}v_o\sigma}{\sqrt{2\pi mkT}} = \sigma d\nu\ exp\left[-\frac{3\phi}{kT}\right]$$

Wilson – Frenkel growth rate

- **Growth rate v**

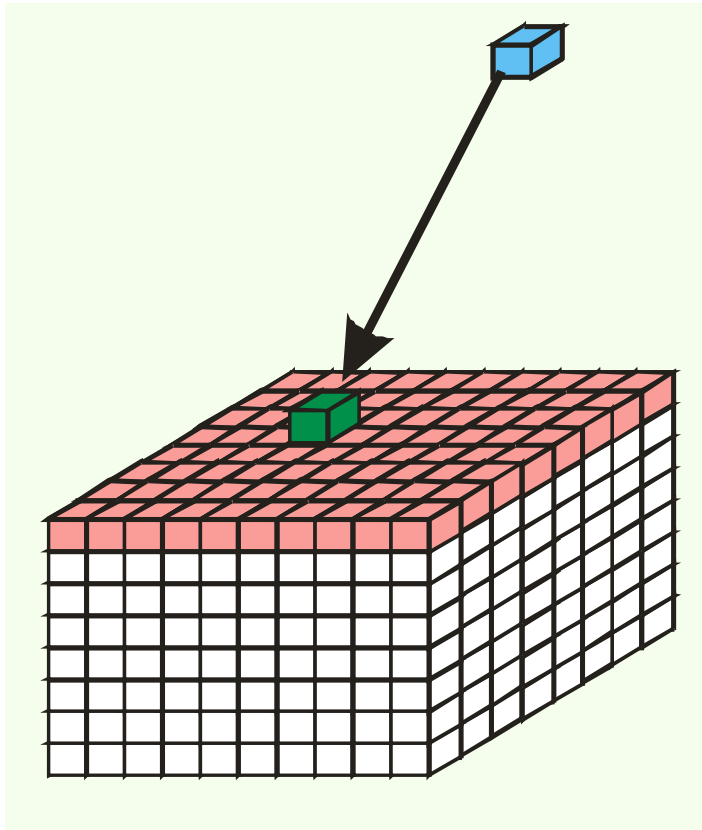$$v = \frac{p_{eq}v_o\sigma}{\sqrt{2\pi mkT}} = v d\sigma \, exp\left[-\frac{3\phi}{kT}\right]$$

$$v = 0.66 \, cm/s$$
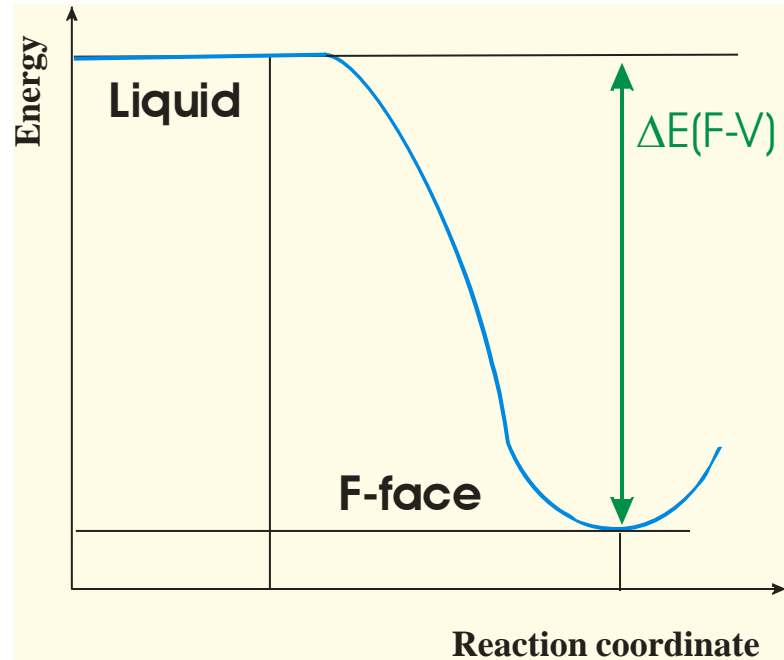
This is the upper bound for crystal growth velocity at:
i/ σ = 0.1
ii/ p = 1 bar = 10^5 N/m²
iii/ T = 298.15 K
iv/ m = m_{N2} = 28 * 1.66 * 10^{-27} kg

Adsorption – F surface (Kossel model)

- **Atomic adsorption at F surface (atomically flat)**

$$E_F = -\phi$$

$$E_V = 0$$

Adsorption – at flat (F) surface

- **Adsorption – site attachment rate R_{ads}**

$$R_{ads} = I_{ads}\varsigma = \frac{p\varsigma}{\sqrt{2\pi mkT}} = n\varsigma\sqrt{\frac{kT}{2\pi m}}$$

- **Desorption – detachment rate R_{des}**

$$R_{des} = \nu c_{sur}(p)exp\left[-\frac{\phi}{kT}\right]$$

- **F surface – vapor – crystal equilibrium (zero supersaturation) – fluxes are balanced I_{tot}**

$$I_{tot} = \frac{p_{eq}}{\sqrt{2\pi mkT}} - \frac{\nu c_{sur-eq}}{\varsigma}\,exp\left[-\frac{\phi}{kT}\right] = 0$$

- **Site occupation (coverage) c_{sur-eq}**

$$c_{sur-eq} = exp\left[-\frac{2\phi}{kT}\right]$$

Adsorption – at flat (F) surface- equilibrium with the crystal (K face)

- **F surface – vapor in equilibrium with the crystal (K surface) – fluxes are balanced**

$$I_{tot} = \frac{p_{eq}\varsigma}{\sqrt{2\pi mkT}} - \nu c_{sur-eq}\, exp\left[-\frac{\phi}{kT}\right] = 0$$
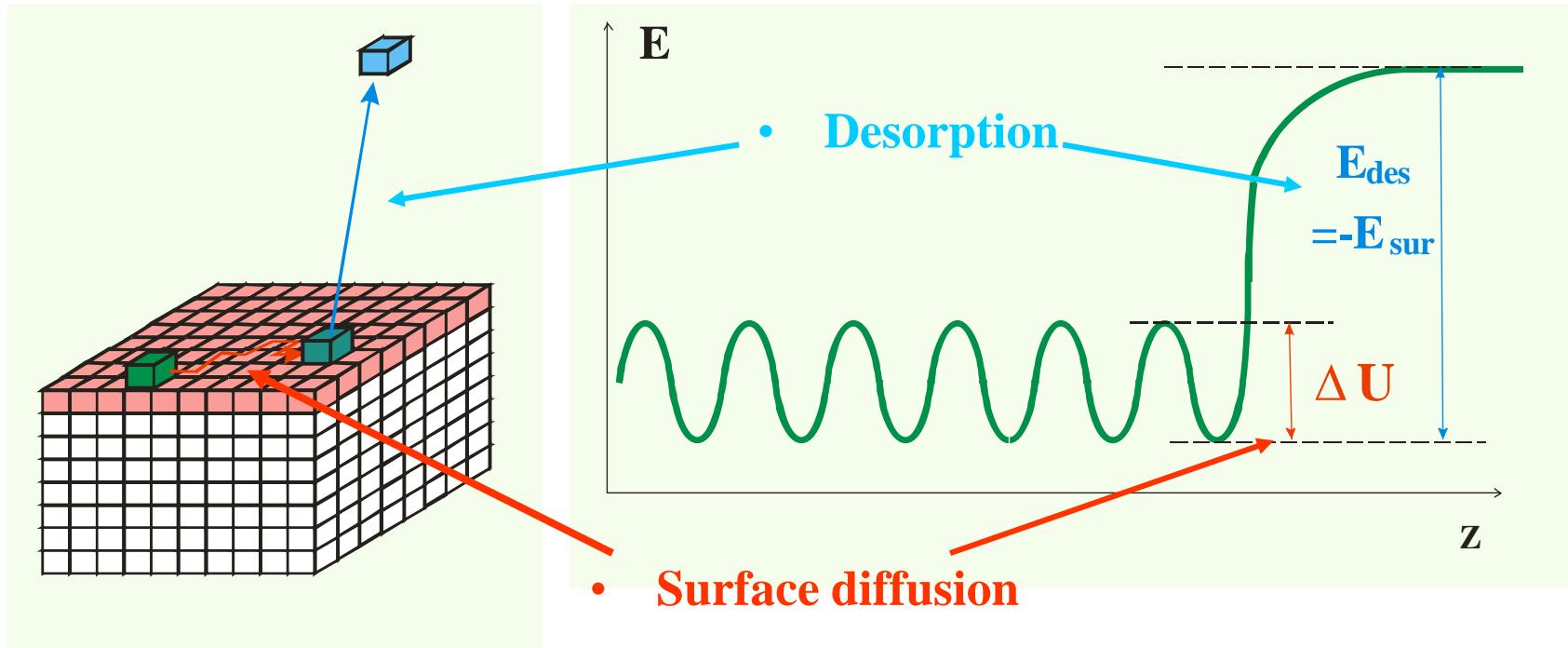
- **Equilibrium with K face – flux and pressure**

$$R_{eq} = \nu\, exp\left[-\frac{3\phi}{kT}\right]$$

- **Site occupation in equilibrium (zero supersaturation) with the crystal and with vapor:**

$$c_{sur-eq} = exp\left[-\frac{2\phi}{kT}\right]$$

Flat surface – surface diffusion and desorption

- **Surface diffusion coefficient depends on the jump barrier U:**

$$D = \frac{a^2 \nu}{4} exp\left[-\frac{\Delta E_{diff}}{kT}\right] = \frac{a^2 \nu}{4} exp\left[-\frac{U}{kT}\right]$$

Flat (F) surface - desorption

- **Desorption – detachment rate R_{des}**

$$R_{des} = \nu \, exp\left[-\frac{E_{des}}{kT}\right] = \nu \, exp\left[-\frac{\phi}{kT}\right]$$

- **Surface residence time τ_{sur} – detachment rate**

$$\tau_{sur} = \frac{1}{\nu} exp\left[\frac{E_{des}}{kT}\right] = \tau_o exp\left(\frac{\phi}{kT}\right)$$

The latter dependence is described for Kossel crystal F surface i.e. for $E_{des} = \phi$.

F surface – surface diffusion length

- **Surface diffusion lenght x_{sur} is determined using surface residence time τ_{sur}**

$$\langle (\Delta r)^2 \rangle = 4 D_{sur} t = 4 D_{sur} \tau_{sur}$$
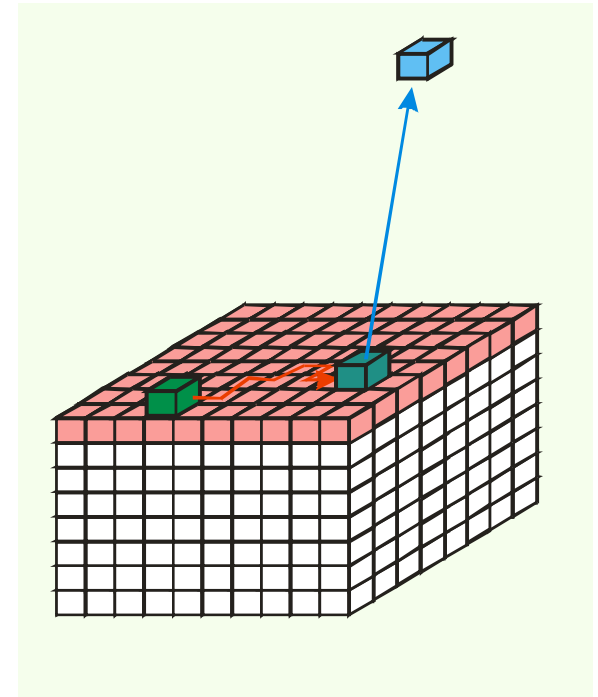
- **Surface diffusion coefficient**

$$D_{sur} = \frac{a^2 \nu}{4} exp\left[-\frac{U}{kT} \right]$$

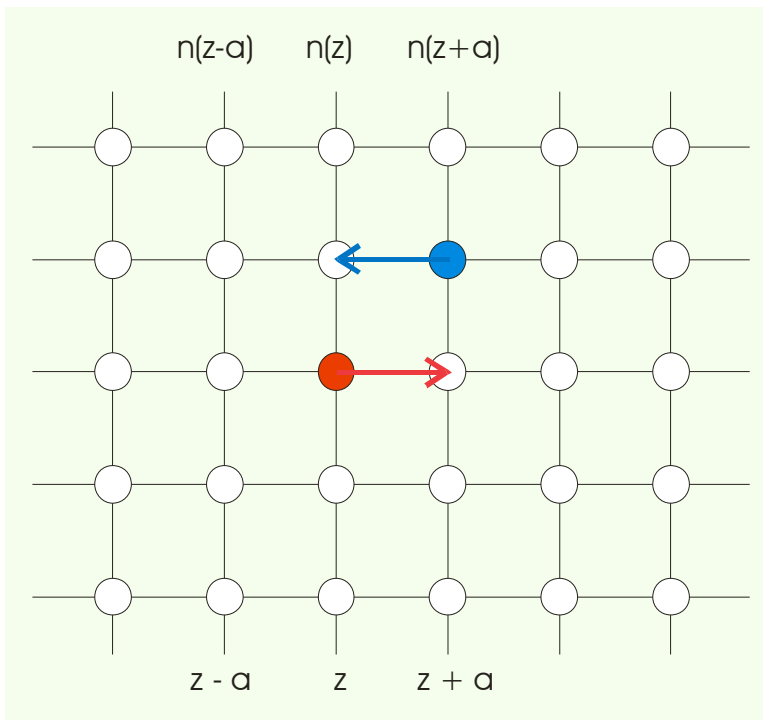- **Surface diffusion length x_{sur} is**

$$x_{sur} = \frac{a}{2} exp\left(-\frac{E_{des} - \Delta E_{diff}}{2kT} \right)$$

- **Kossel crystal**

$$x_{sur} = \frac{a}{2} exp\left(\frac{\phi - U}{2kT} \right)$$

Surface diffusion – chemical diffusion

- **Average density of adsorbed species changes slowly in space:**

$$n_{sur}(z + a) = n_{sur} + \delta n_{sur}$$

$$n_{sur}(z - a) = n_{sur} - \delta n_{sur}$$

i.e. over the distance a the change is small, and could be approximated by linear term.

- **The velocity of the jump of the species to the right and left is**

$$R(z \rightarrow z + a) = n_{sur}(z)\nu P(z \rightarrow z + a)$$

$$R(z + a \rightarrow z) = n_{sur}(z + a)\nu P(z + a \rightarrow z)$$

Surface diffusion (chemical) - Fick law

- **Effective flux is the difference of these jumps**

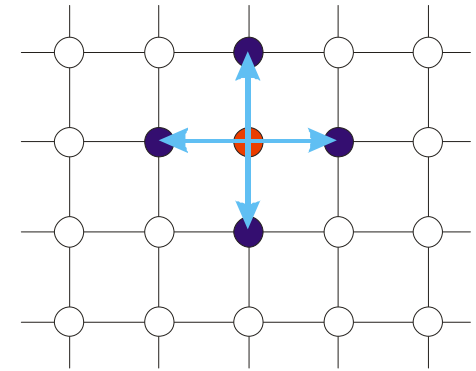$$j_{sur} = R(z \to z + a) - R(z + a \to z)$$

- **Linear expansion gives**

$$j_{sur} = -\left(\frac{dn_{sur}}{dz}\right) a \, \nu P(z \to z + a)$$

$$P(z \to z + a) = \frac{1}{4}$$

$$j_{sur} = -\frac{a^2 \nu}{4}\left(\frac{dn_{sur}}{dz}\right) = -D_{sur}\left(\frac{dn_{sur}}{dz}\right)$$

- **General case – Fick law**

$$\vec{j}_{sur} = -D_{sur}\nabla n_{sur} = -D_{sur}\eta\nabla c_{sur}$$

c_{sur} – coverage, i.e. fraction of sites that are occupied

Adsorbate dynamics

- **Mass conservation law (next lecture – derivation)**

$$\nabla \cdot \vec{j}_{sur} + I = 0$$

- **Mass source for unit of surface area I – adsorption and desorption balance**

$$I = I_{ads} - I_{des}$$

- **Adsorbate dynamics equation**

$$D_{sur}\Delta n_{sur} = I_{ads} - I_{des}$$

Sources – adsorption and desorption

- **Vapor mass sources are calculated using vapor supersaturation σ_V:**

$$\sigma_V = \frac{p - p_{eq}}{p_{eq}}$$

- **Surface supersaturation, different from vapor supersaturation is;**

$$\sigma = \frac{n_{sur} - n_{sur-eq}}{n_{sur-eq}} = \frac{c_{sur} - c_{sur-eq}}{c_{sur-eq}}$$

- **Adsorbate density:**

$$n_{sur} = \eta c_{sur} = \frac{c_{sur}}{\varsigma}$$

- **Total source is:**

$$I = I_{ads} - I_{des} = \frac{p}{\sqrt{2\pi m k T}} - \frac{\nu c_{sur}}{\varsigma} \, exp\left[-\frac{\phi}{kT}\right]$$

Adsorbate dynamics - sources

- **Laplace term is:**

$$D_{sur}\Delta n_{sur} = D_{sur}\eta\Delta c_{sur} = \frac{D_{sur}\Delta(c_{sur} - c_{sur-eq})}{\varsigma} = \frac{D_{sur}c_{sur-eq}\Delta\sigma}{\varsigma}$$

- **Vapor mass source is:**

$$I_{ads} = \frac{p}{\sqrt{2\pi mkT}} = \frac{\nu\sigma_V c_{sur-eq}}{\varsigma} \, exp\left[-\frac{\phi}{kT}\right] = \frac{\sigma_V c_{sur-eq}}{\varsigma\tau_{sur}}$$

- **Desorption escape is:**

$$I_{des} = \frac{\nu c_{sur}}{\varsigma} \, exp\left[-\frac{\phi}{kT}\right] = \frac{\nu\sigma c_{sur-eq}}{\varsigma} \, exp\left[-\frac{\phi}{kT}\right] = \frac{\sigma c_{sur-eq}}{\varsigma\tau_{sur}}$$

Adsorbate dynamics - sources

- **The basic equation is:**

$$\frac{D_{sur}c_{sur-eq}\Delta\sigma}{\varsigma} = \frac{\sigma c_{sur-eq}\tau_{sur}}{\varsigma} - \frac{\sigma_V c_{sur-eq}\tau_{sur}}{\varsigma}$$

- **Transferred to:**

$$\frac{D_{sur}\Delta\sigma}{\tau_{sur}} = \sigma - \sigma_V$$

- **The fundamental equation is:**

$$x_{sur}^2\Delta\sigma = \sigma - \sigma_V$$

Vicinal (V) surfaces – steps and terraces

Surfaces consisting of steps and flat areas between (terraces) we call vicinal surfaces.
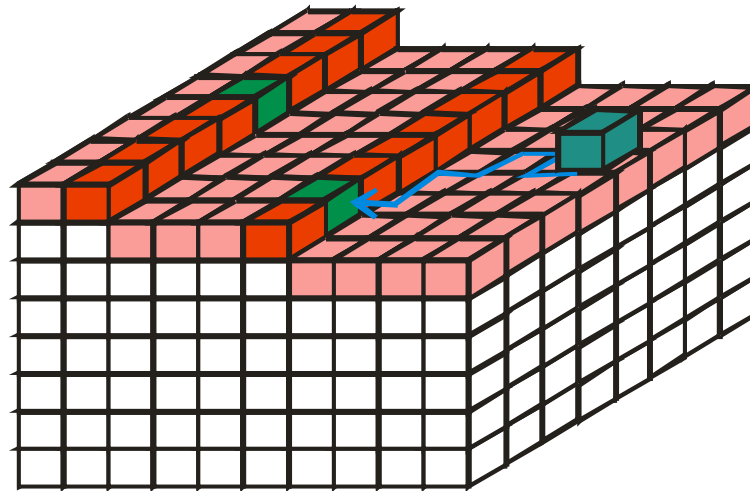
- **The fundamental equation is governing adsorbate dynamics at terraces:**

$$x_{sur}^2 \Delta\sigma = \sigma - \sigma_V$$

- **Steps – boundary conditions?**

Energy of the steps
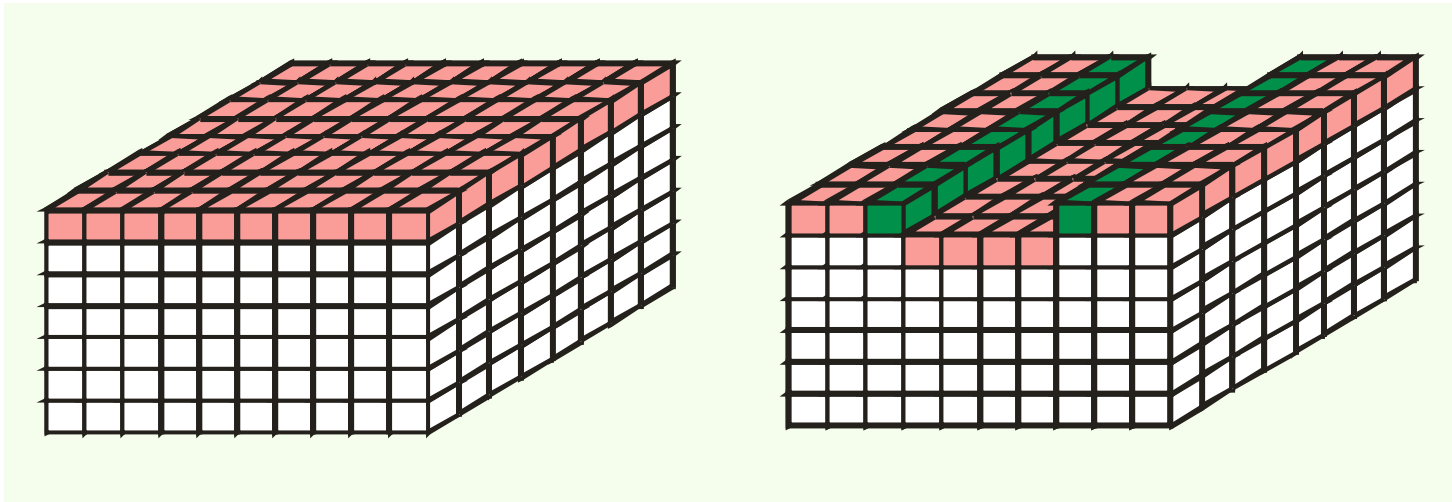
- **Atom row escape - creation of two steps**
- **Atom escape - two bonds are broken**
- **Step energy in a distance – half of the bond energy**

$$\gamma = \frac{\phi}{2}$$

Step- kink creation
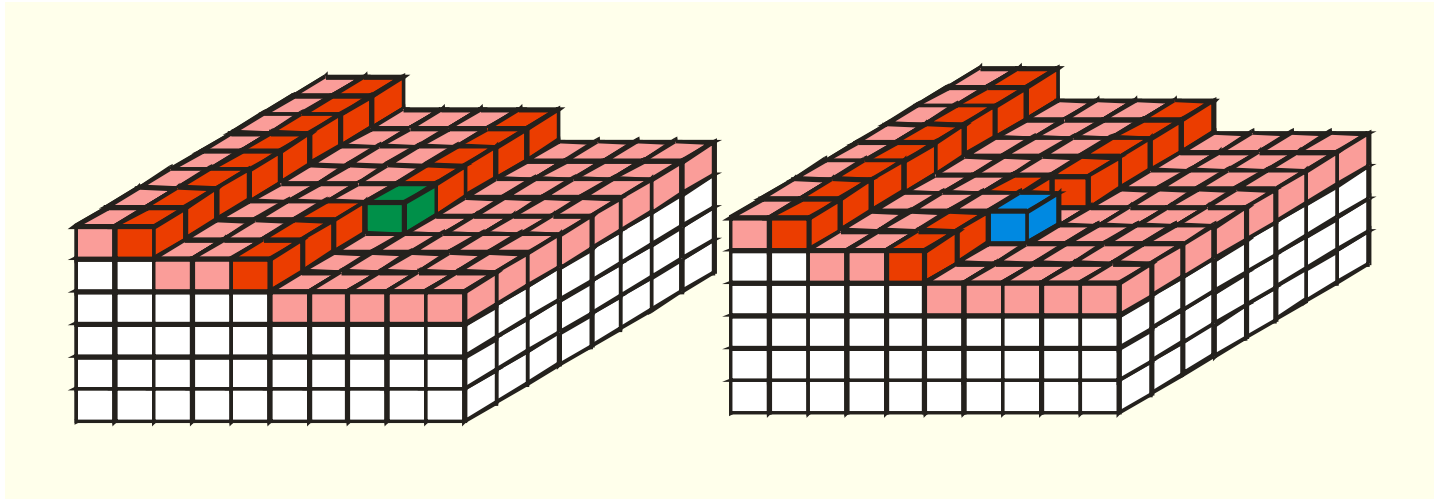
- **Atom escape - creation of two kinks**
- **Atom escape - two bonds are broken**
- **Kink energy – half of the bond energy**

Microscopic structure of the steps

- **Atom escape - creation of two kinks**
- **Atom escape - two bonds are broken**
- **Kink concentration at the step:**

$$c_{kink} = P\left(\frac{E_{kink}}{2}\right) = exp\left(-\frac{\phi}{2kT}\right)$$

- **Kink concentration at the step is strongly temperature dependent. Nevertheless in typical growth experiment it is high. The distance between two kinks is**

$$l_{kink} = \frac{a}{2c_{kink}} = \frac{a}{2}exp\left(\frac{\phi}{2kT}\right)$$

Steps – linear growth sources Burton, Cabrera Frank (BCF) model

Atoms/molecules are incorporated at the steps from:

- **Diffusion at terraces**

- **From the vapor directly**

W.K. Burton, N. Cabrera & F.C. Frank, Philosph. Trans. Roy. Soc. (London) A243 (1951) 299
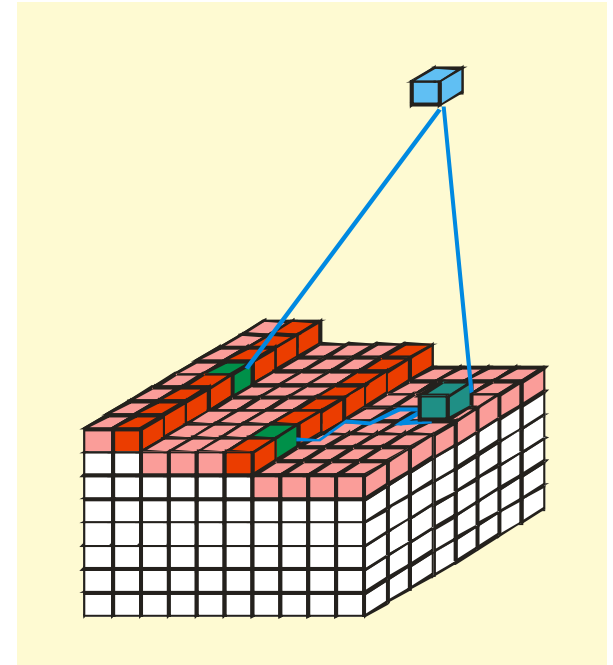
Symmetry of the steps

- **Symmetric steps - jump barrier from upper and lower are identical**
- **Asymmetric steps - jump barrier from upper and lower are different**
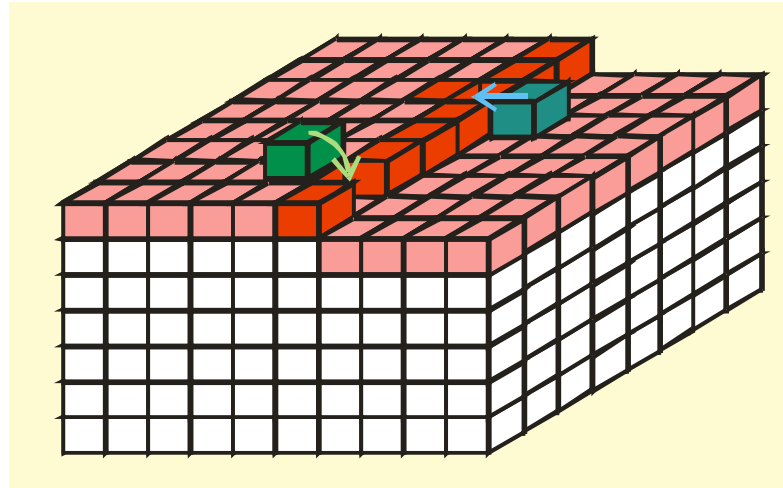
- **Step asymmetry – Schwoebel barrier**

R.L. Schwoebel & E.J. Shipsey, J. Appl. Phys. 37 (1966) 3682

R.L. Schwoebel, J. Appl. Phys. 40 (1969) 614

Symmetric steps – simple model dynamics

- **Jumps are equally probable from upper and lower terrace**
- **No additional energy barrier for terrace-step jump**
- **Simple model – supersaturation at the step vanishes**

- **Terrace dynamics**

$$x_{sur}^2 \Delta\sigma = \sigma - \sigma_V \qquad x_{sur}^2 \frac{d^2\sigma}{dz^2} = \sigma - \sigma_V$$

- **Boundary condition at the step**

$$\sigma = 0 \qquad z = 0$$

- **Solution**

$$\sigma = \sigma_V[1 - exp(-z/x_{sur})] \qquad z > 0$$

$$\sigma = \sigma_V[1 - exp(z/x_{sur})] \qquad z < 0$$
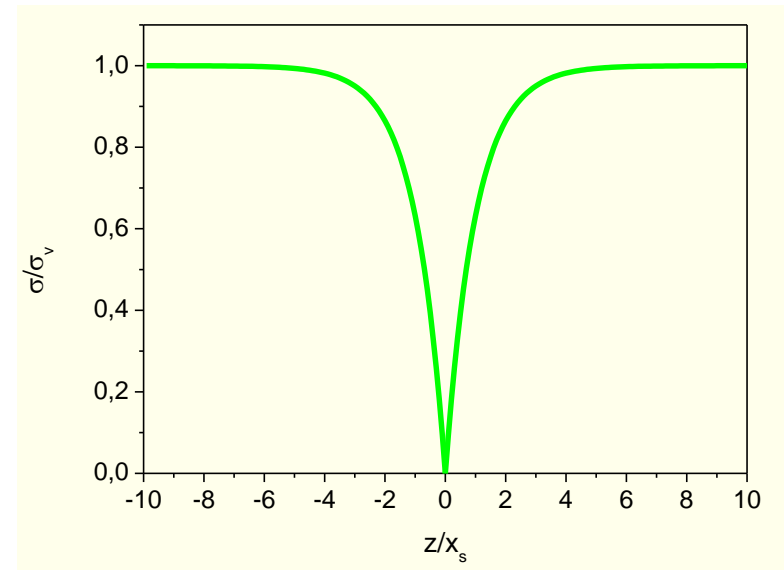
Motion of single, symmetric step

- **Adatom flux to the step I_{step}**

$$I_{step} = 2j_{sur}(z = 0) = 2D_{sur}\eta c_{sur-eq}\nabla\sigma = \frac{2D_{sur}\eta c_{sur-eq}\sigma_V}{x_{sur}}$$
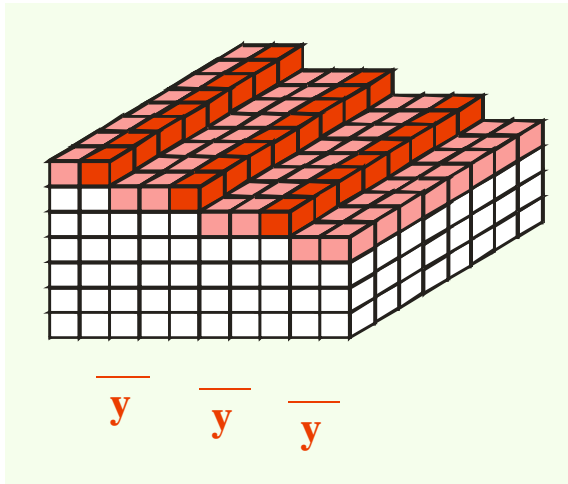
- **Step site atom attachment frequency R_{step}**

$$R_{step} = I_{step}a = \frac{2D_{sur}\eta c_{sur-eq}\sigma_V a}{x_{sur}}$$

- **Step velocity v_{step}**

$$v_{step} = R_{step}a = \frac{2D_{sur}\eta c_{sur-eq}\sigma_V a^2}{x_{sur}} = \frac{2x_{sur}c_{sur-eq}\sigma_V}{\tau_{sur}}$$

Parallel equidistant symmetric steps - dynamics

- **Step distance y:**

$$\overline{y} \quad \overline{y} \quad \overline{y}$$
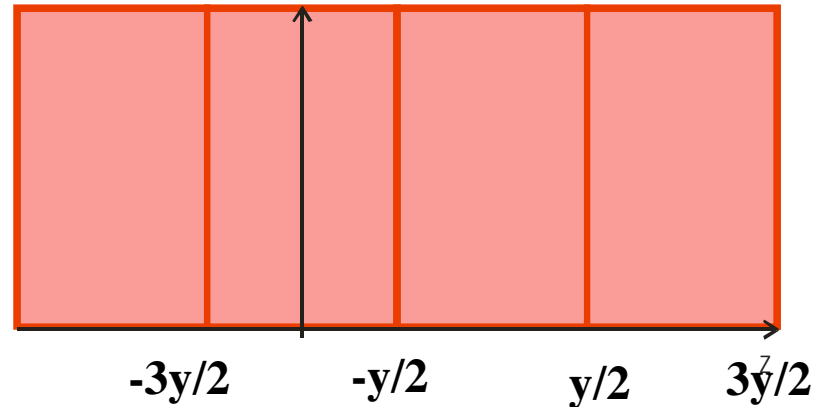
- **Terrace dynamics**

$$x_{sur}^2 \frac{d^2\sigma}{dz^2} = \sigma - \sigma_V$$

- **Boundary condition at the step**

$$\sigma = 0 \qquad z = \pm\frac{y}{2}, \pm\frac{3y}{2}, \frac{5y}{2}, \dots,$$

$$-3y/2 \qquad -y/2 \qquad y/2 \qquad 3y/2$$

- **Solution (single terrace)**

$$\sigma = \sigma_V \left[1 - \frac{csh(z/x_{sur})}{csh(y/2x_{sur})} \right]$$

- **Periodicity (multiple terraces)**

$$\sigma = \sigma_V \left[1 - \frac{csh[(z - 2ny)/x_{sur}]}{csh(y/2x_{sur})} \right]$$

$$n = 0, \pm1, \pm2, \pm3, \dots$$

Parallel equidistant symmetric steps - supersaturation

$$\sigma = \sigma_V \left[1 - \frac{csh(z/x_{sur})}{csh(y/2x_{sur})} \right] \qquad -\frac{y}{2} \leq z \leq \frac{y}{2}$$
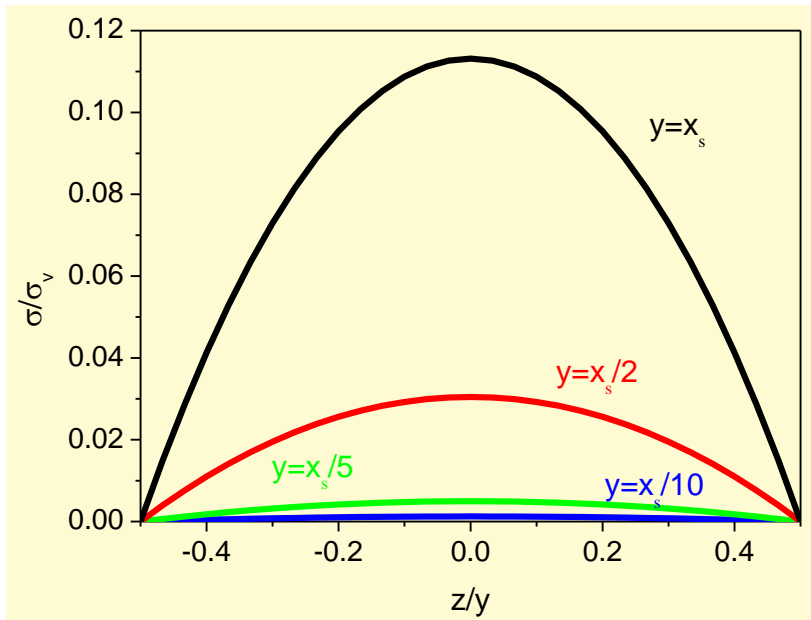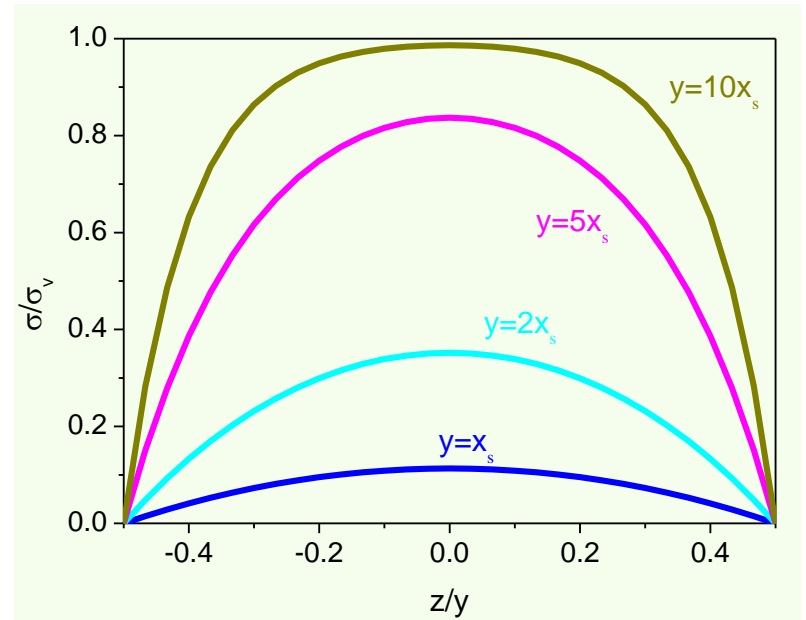
$$y \leq x_{sur} \qquad\qquad y \geq x_{sur}$$

Parallel equidistant symmetric steps – step motion

- **Adatom flux to the step I_{step}**

$$I_{step} = 2D_{sur}\eta c_{sur-eq}\nabla\sigma = \frac{2D_{sur}\eta c_{sur-eq}\sigma_V}{x_{sur}}\tanh\left(\frac{y}{2x_{sur}}\right)$$

- **Step site atom attachment frequency R_{step}**

$$R_{step} = I_{step}a = \frac{2D_{sur}\eta c_{sur-eq}\sigma_V a}{x_{sur}}\tanh\left(\frac{y}{2x_{sur}}\right)$$

- **Steps velocity $v_{step}(\text{y})$**

$$v_{step}(y) = R_{step}a = \frac{2x_{sur}c_{sur-eq}\sigma_V}{\tau_{sur}}\tanh\left(\frac{y}{2x_{sur}}\right)$$

Parallel symmetric steps – step motion – distance dependence

- **Parallel steps velocity $v_{step}(y)$**

$$v_{step}(y) = \frac{2x_{sur}c_{sur-eq}\sigma_V}{\tau_{sur}} \tanh\left(\frac{y}{2x_{sur}}\right)$$
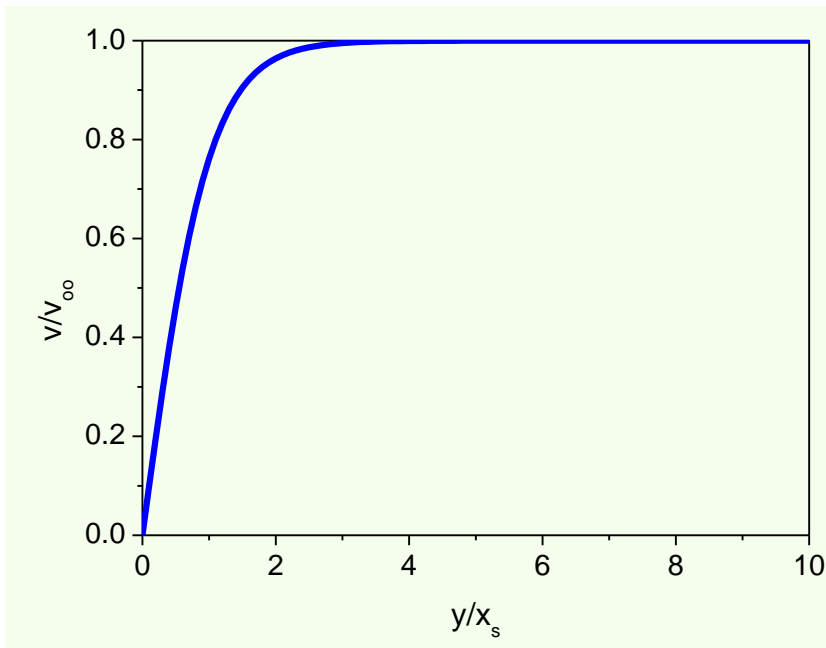
- **Single step velocity $v_{\infty} = v_{step}(\infty)$**

$$v_{step}(\infty) = \frac{2x_{sur}c_{sur-eq}\sigma_V}{\tau_{sur}}$$

Small interstep distance – low velocity – diffusional repulsion of the steps

Asymmetric steps – simple model dynamics

- **Jumps from upper and lower terrace have different barriers**
- **Simple model – supersaturation at the step vanishes**

- **Terrace dynamics**

$$x^2_{sur}\Delta\sigma = \sigma - \sigma_V \qquad x^2_{sur}\frac{d^2\sigma}{dz^2} = \sigma - \sigma_V$$

- **Boundary condition at the step – lower terrace**

$$\sigma = 0 \qquad z = 0_+$$

- **Boundary condition at the step – upper terrace**

$$j = 0 \qquad z = 0_-$$

- **Solution**

$$\sigma = \sigma_V[1 - exp(-z/x_{sur})] \qquad z > 0$$

$$\sigma = \sigma_V \qquad z < 0$$
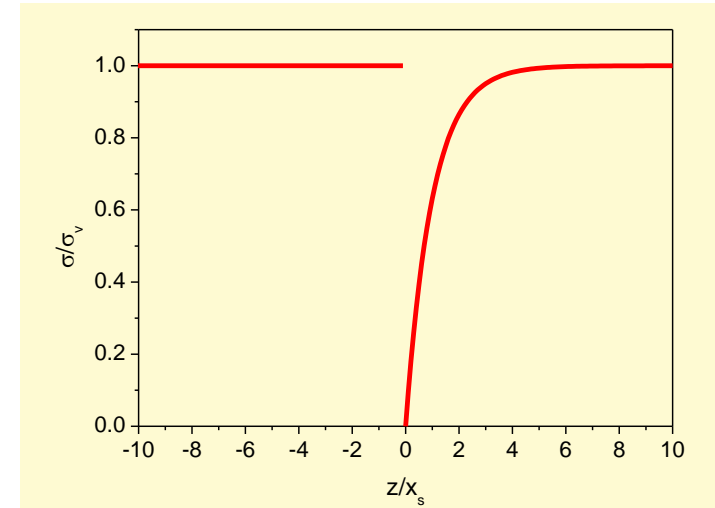
Parallel asymmetric steps - macrosteps

- **Macrostep – stack of many atomic steps, at the same position**
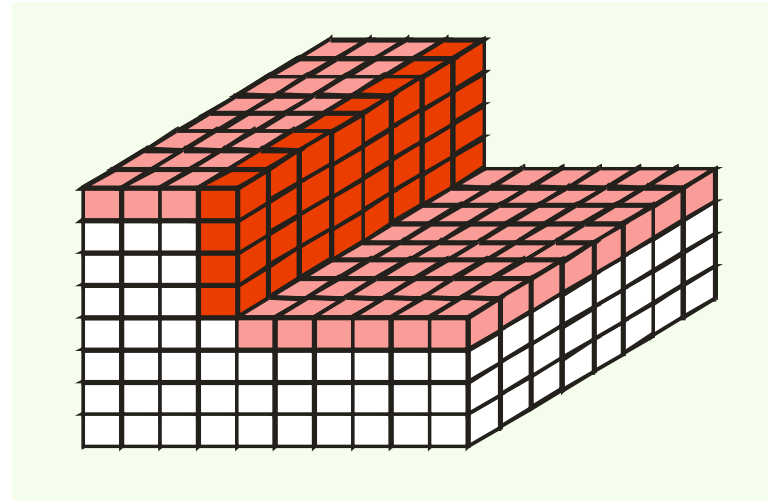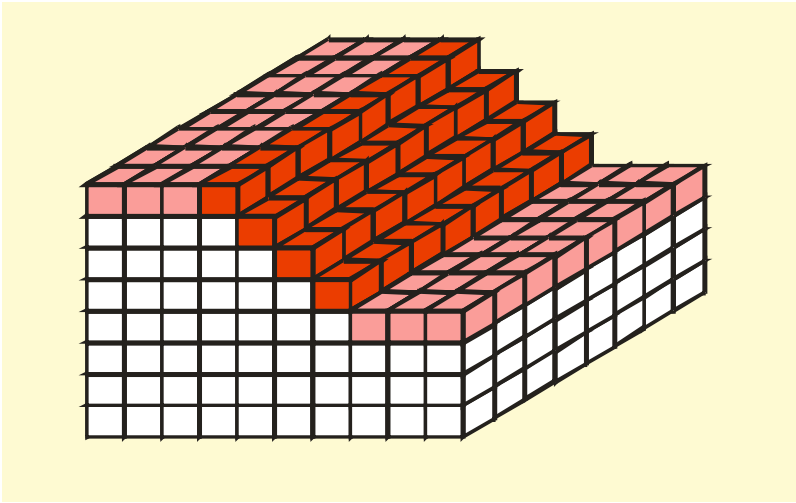
- **Macrosteps – coalescence of a number of atomic steps**
- **Symmetric steps repel each other – no macrosteps creation tendency**
- **This conclusion is independent of the step orientation**
- **Additional factor can contribute to macrostep formation – impurities, kink poisoning, etc.**

Steps – sources and geometry

Sources

- **Surface preparation for epitaxy**
- **Two-dimensional nucleation**
- **Screw dislocation – continuous step source**
- **Other extended defects, such as stacking faults**

Geometry

- **Microstructure – equilibrium**
- **Macrostructure – step shape – purely nonequilibrium, history dependent**
- **Step height - purely nonequilibrium, history dependent**

Steps created by surface preparation

- **Surface misorientation – small angle to low Miller indices surface e.g. 2°
  on GaN surface**
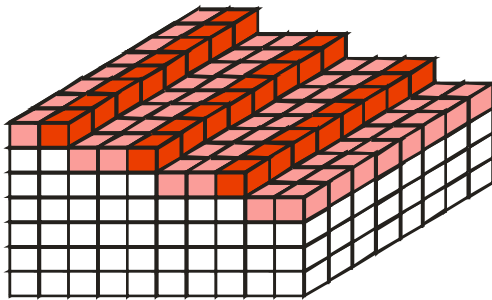
Sample size l = 5 mm

Layer thickness – d

$$d = l \, tg(\alpha) = 172 \, \mu m$$

**This is more than sufficient for typical optoelectronic device where
the structure is 1 micron thick**

Step source – 2-d nucleation

- **Fluctuation – emergence of finite island of a new atomic layer:**
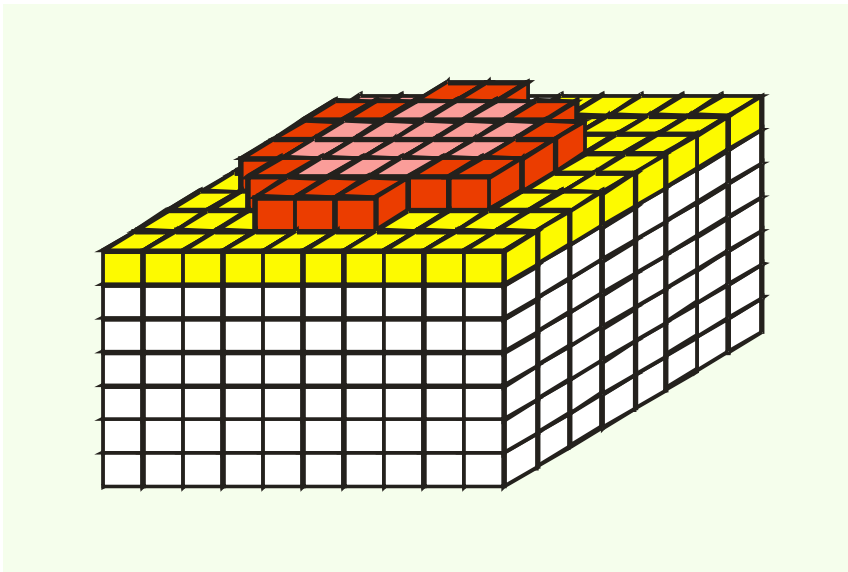
- **Edge energy of the length a - γ**

- **Single site area $s_o = a^2$**

- **Supersaturation**

$$\sigma = \frac{\Delta\mu}{kT}$$

Nucleation energy barrier

- **Total energy change in island creation:**

$$\Delta E = \frac{\pi r^2 \Delta\mu}{a^2} + \frac{2\pi r\gamma}{a}$$

- **Positive term is due to edge energy, $\gamma > 0$**

- **Negative term is due to bulk energy, $\Delta\mu < 0$**
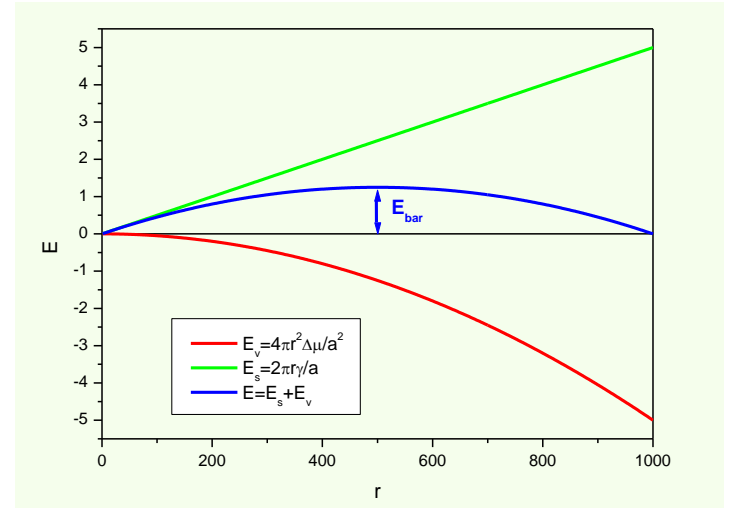
- **Positive term dominates at small r, negative for large r, the maximum energy, at critical radius is called nucleation barrier**

$$r_{crit} = -\frac{\gamma a}{\Delta\mu} = \frac{\gamma a}{|\Delta\mu|} = \frac{\gamma a}{kT\sigma} = \frac{\phi a}{2|\Delta\mu|} \qquad \gamma = \frac{\phi}{2}$$

- **Nucleation barrier**

$$\Delta E_{bar} = \frac{\pi\gamma^2}{|\Delta\mu|} = \frac{\pi\phi^2}{4|\Delta\mu|}$$

2-d nucleation rate (Zeldowich)

- **Nucleation rate – attempt frequency ν, number of sites at circumference N_s, and transition probability**

$$R_{2d-nucl} = \nu\, n_c\, P = \nu\, \frac{2\pi r_{crit}}{a}\, exp\left(-\frac{\Delta E_{bar}}{kT}\right)$$

- **Nucleation rate – for single nucleus**

$$R_{2d-nucl} = \frac{2\pi\nu\gamma}{|\Delta\mu|}\, exp\left(-\frac{\pi\gamma^2}{kT|\Delta\mu|}\right) = \frac{\pi\nu\phi}{|\Delta\mu|}\, exp\left(-\frac{\pi\phi^2}{4kT|\Delta\mu|}\right)$$

- **Nucleation rate – for crystal surface A**

$$\gamma = \frac{\phi}{2}$$

$$R_{2d-nucl}(A) = R_{2d-nucl}\frac{A}{\pi r_{crit}^2} = \frac{2\nu A|\Delta\mu|}{\gamma a^2}\, exp\left(-\frac{\pi\gamma^2}{kT|\Delta\mu|}\right)$$

$$R_{2d-nucl}(A) = \frac{\nu A|\Delta\mu|}{\phi a^2}\, exp\left(-\frac{\pi\phi^2}{4kT|\Delta\mu|}\right)$$

2-d nucleation rate – supersaturation dependence

- **Nucleation rate at the surface area A:**

$$R_{2d-nucl}(A) = \frac{\nu A |\Delta\mu|}{\phi a^2} \; exp\left(-\frac{\pi\phi^2}{4kT|\Delta\mu|}\right)$$
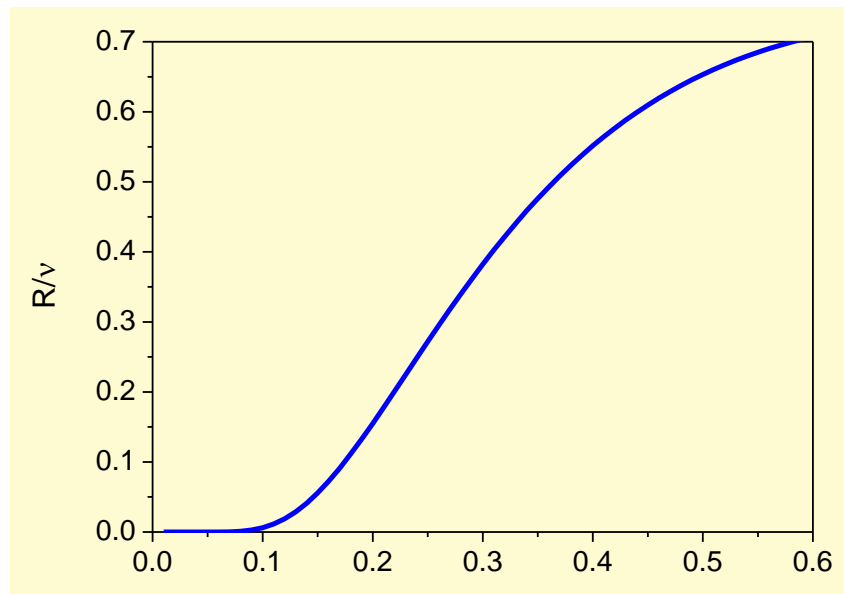
2-d nucelation – macrostep generation

- **2-d nucleation could be fast as compared to layer completion, - creation of macrosteps is possible**
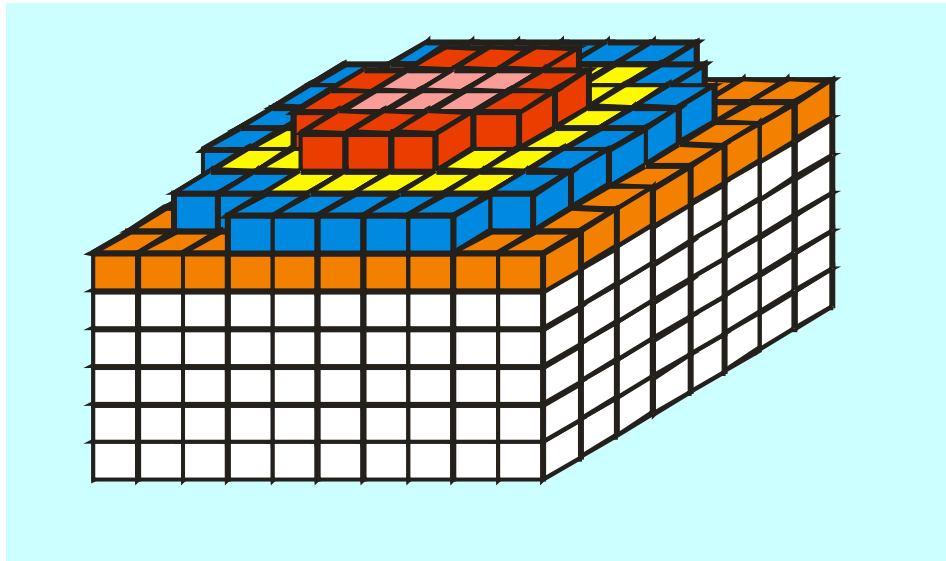
Growth – size increase of concentric circles

Growth of concentric circles

- **Supersaturation at curved step is reduced by Gibbs-Thomson effect:**

$$\sigma(r) = \sigma \left(1 - \frac{r_{crit}}{r} \right)$$

r_{crit} – **critical nucleation radius (2d)**

$$r_{crit} = -\frac{\gamma a}{\Delta \mu} = \frac{\gamma a}{|\Delta \mu|} = \frac{\gamma a}{kT\sigma} = \frac{\phi a}{2|\Delta \mu|}$$

- **Velocity $v_{step}(y)$ of the curved steps, separated by the distance y:**

$$v_{step}(y) = \frac{2D_{sur}\eta c_{sur-eq}\sigma_V a^2}{x_{sur}} \tanh\left(\frac{y}{2x_{sur}} \right)\left(1 - \frac{r_{crit}}{r} \right)$$

Step sources – screw dislocations

- **Dislocation atomic structure**
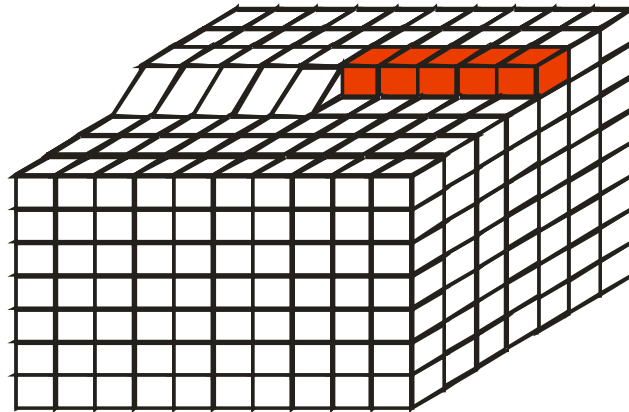
- **In equilibrium step is straight line.**

Supercooling – step is curved

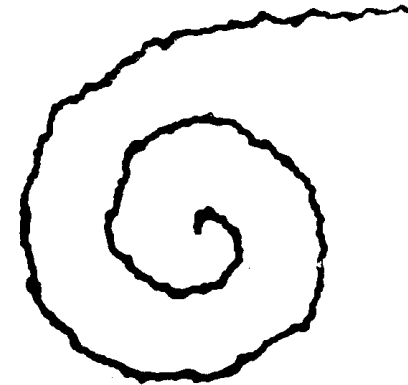- **High temperature – Archimedean spiral**
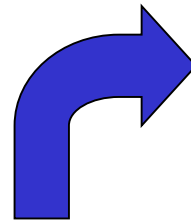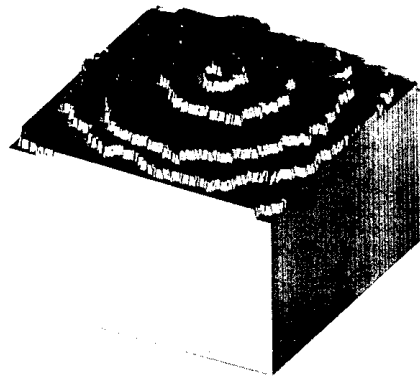
High supersaturation
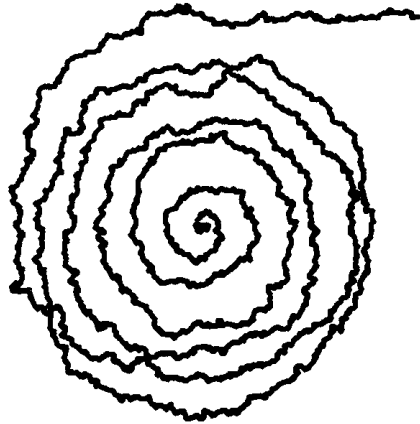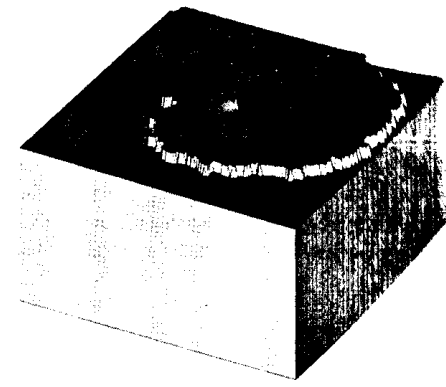
$$\sigma = \frac{\Delta\mu}{kT} = 0.4$$
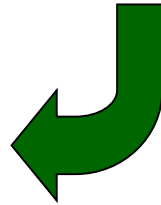
Low supersaturation

(a)

$$\sigma = \frac{\Delta\mu}{kT} = 0.1$$

R. H. Swendsen et al. J. Cryst. Growth 35 (1976) 73

Step motion – spiral rotation – Archimedean spiral

- **Step velocity depends on the curvature:**

$$v_{step}(r) = v_{step}\left(1 - \frac{r_{crit}}{r}\right)$$

- **Approximate solution- Archimedean spiral:**

$$r(\theta) = 2r_{crit}\theta$$

- **Angular velocity is:**

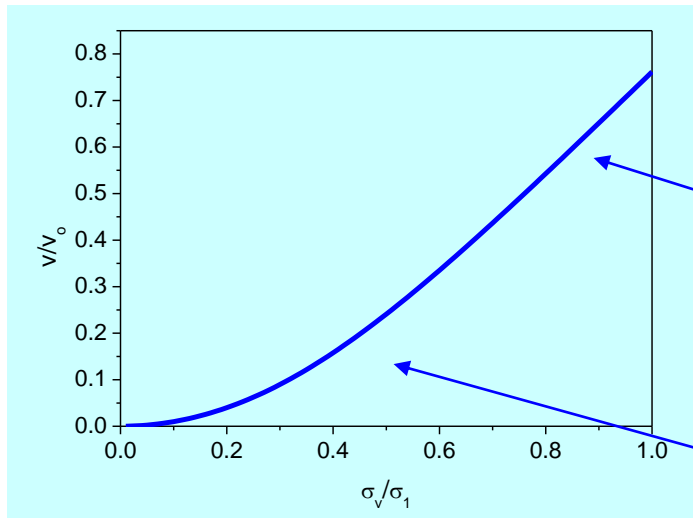$$\omega = \frac{v_{step}}{2r_{crit}}$$

- **Distance between spiral arms is:**

$$y_o = 2\pi r_{crit}$$

- **Spiral shape is function of the supersaturation**
- **Spiral velocity depends on the surface diffusion rate**

Spiral dislocation controlleed growth rate

- **Growth rate is equal to the angular velocity ω multiplied by layer thickness a:**

$$v = \frac{\omega a}{2\pi} = \frac{v_{step}a}{2r_{crit}} = \frac{a\sigma_V^2}{\sigma_1\tau_{sur}} \tanh\left(\frac{\sigma_1}{\sigma_V}\right) \cong \begin{cases} \dfrac{a\sigma_V^2}{\sigma_1\tau_{sur}} & \sigma_V \ll \sigma_1 \\ \dfrac{a\sigma_V}{\tau_{sur}} & \sigma_V \gg \sigma_1 \end{cases}$$

$$\sigma_1 = \frac{\pi\gamma a}{2kTx_{sur}} = \left(\frac{a}{x_{sur}}\right)\left(\frac{\pi\phi}{4kT}\right)$$
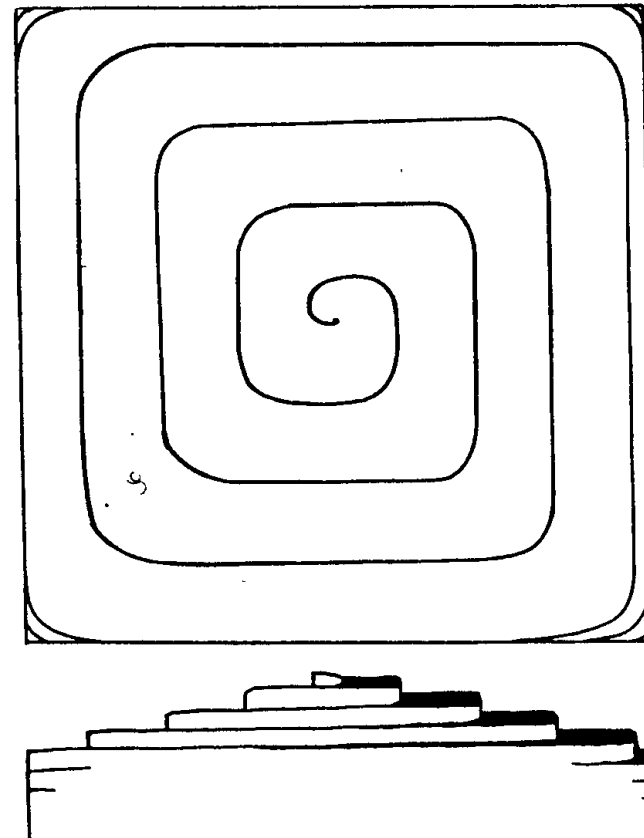
linear

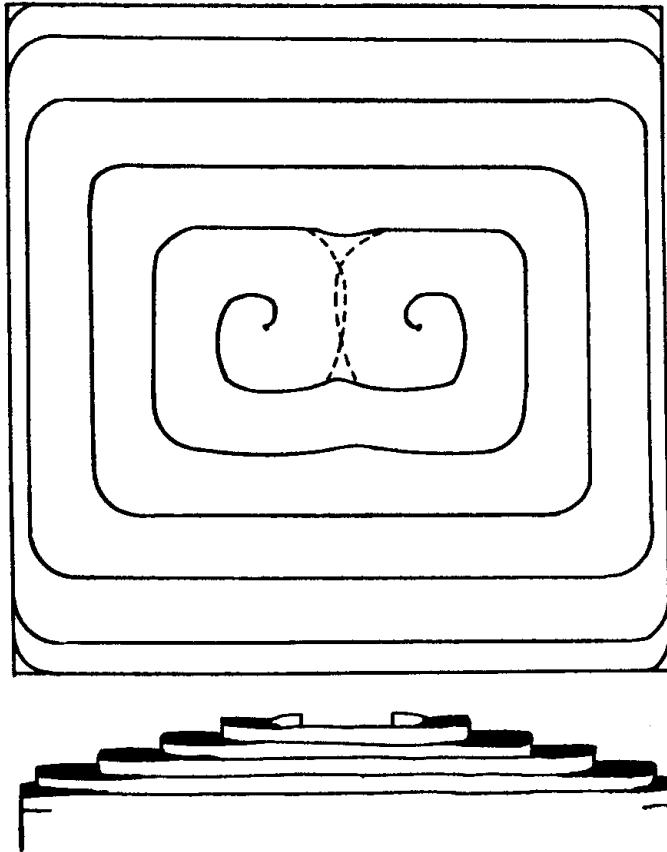**parabolic**

Screw dislocation – growth pyramid.

- **Distance between spiral line is:**

$$y_o = 4\pi r_{crit} =$$

$$\frac{\pi \gamma a}{k_B T \sigma_v} = \frac{\pi \varphi a}{k_B T \sigma_v}$$

$$tg\alpha = \frac{a}{y_o} = \frac{4 k_B T \sigma_v}{\varphi}$$

Two dislocations of the opposite sign
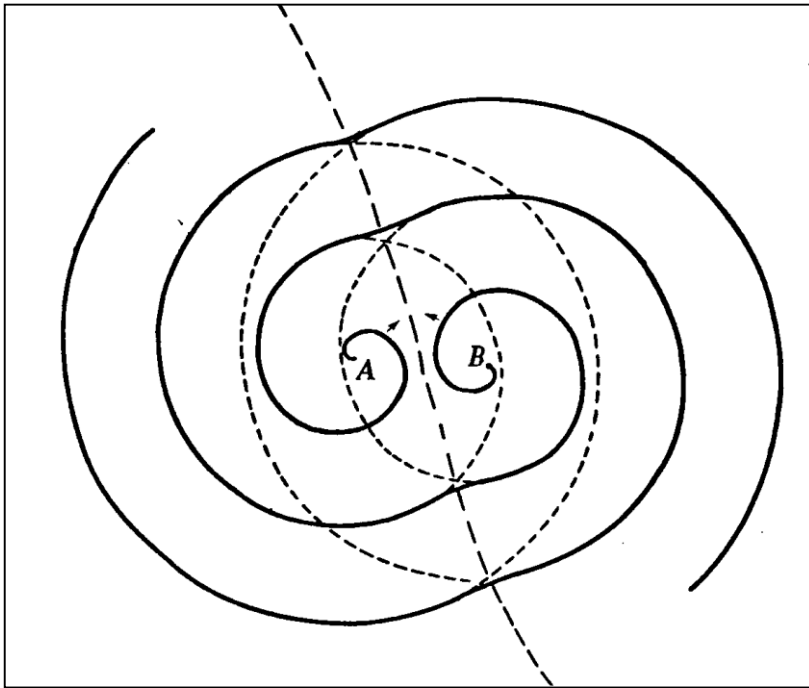

L – distance between dislocations

$$L > r_{crit}$$

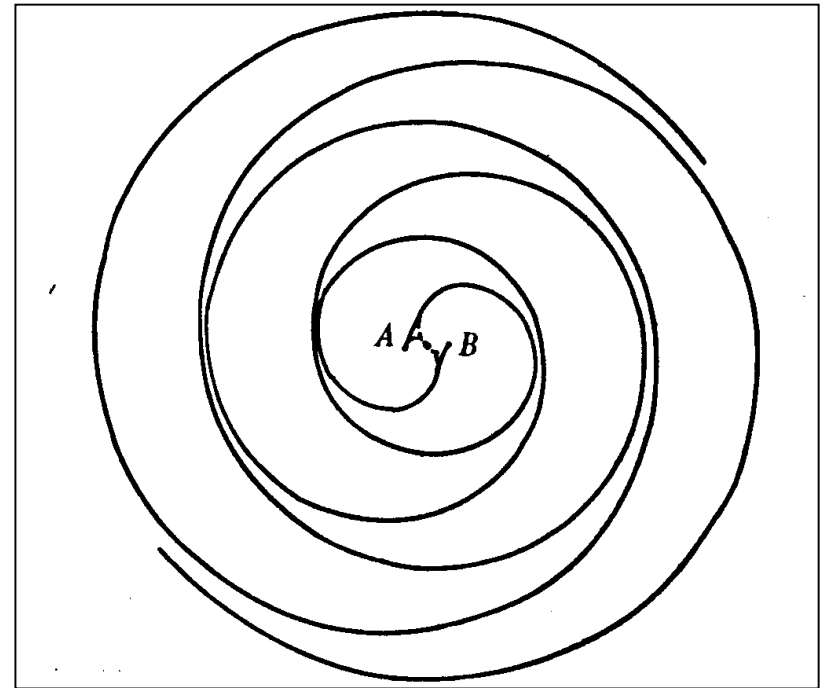$$L < r_{crit}$$

Two dislocation of the same sign

$$L > r_{crit}$$

$$L < r_{crit}$$

Multiple dislocations of the same sign